MATHEMATICS

Key Stage 3

(Years 8, 9 & 10)

By R.M. O'Toole

B.A., M.C., M.S.A.

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First Published in 1996 (Revised in 2006) by:

Mathematics Publishing Company,
'The Cottage'
45, Blackstaff Road,
Clough,
Downpatrick,
Co. Down,
BT30 8SR.

Tel. 028 44 851818 Fax. 028 44 851818

email mathsbooks@hotmail.co.uk

This publication has been deposited with 'The Legal Deposit Office', British Library, Boston Spa, Wetherby, W. Yorkshire, LS 23 7BY.

ISBN No. 1900043 408

Testimonials

Below is a selection of written comments received from people who have used our material.

Key Stage 2 (9-11 year olds)

Extracts from a head teacher's letter:

'… *very well received by parents, teachers and pupils …*'
'… *self contained…*'
'… *highly structured …*'
'… *all children including the less well able are helped …*'
'…*to develop concepts through a series of clearly defined steps …*'
'… *increased confidence for pupils …*'
'… *parents find user friendly as worked examples are given …*'
'… *language and notation are simple and clearly defined …*'

From a 10 year old pupil (boy):

'… *the material describes the working out in a way that is easy. The worked examples are laid out very clearly …*'

GCSE (15-16 year olds)

From a 15 year old pupil (boy):

'… *simple and easy way to learn maths …*'
'… *careful explanations of each topic…*'
'… *also questions to make sure you know and understand what you have learned, and each question has a worked answer to check everything you have covered …*'
'… *so you are never left without any help …*'

From a 15 year old pupil (girl):

'…*easy to understand …*'
'…*clear and concise …*'
'…*thoroughly recommended …*'

GCSE Additional (15-16 year olds)

From a 16 year old pupil (girl):

'… *GCSE Mechanics was very helpful …*'

‘… *clearly explained and easy to understand …’*
‘… *well laid out …’*
‘… *well structured …’*
‘… *I would not hesitate to use these again …’*

From a 16 year old pupil (boy):

‘… *self-explanatory and easy …’*
‘… *laid down basis of skill required …’*
‘… *helped me consolidate …’*
‘… *succinct and effective …’*
‘… *boosted my confidence …’*
‘… *contributed significantly towards helping me to prepare for exams …’*

GCE Advanced (18 year olds)

From a 19 year old university student (man):

‘… *may I put on record my appreciation …’*
‘… *your material… gave me help and reinforcement …’*
‘… *increasing my confidence to pursue my maths …’*
‘… *I am now enjoying life at university …’*

Acknowledgements

Special thanks to Cormac O'Toole, who advised on the computer technical end and to Denise Tumelty for design of covers.

Finally, I would like to thank all our customers for buying our books and for their kind letters of appreciation.

G.B. O'Toole, B.A. (Hons.), CertPFS

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Dedication

I dedicate this work to the memory of my late parents, Jack and Mary Tumelty.

Ros

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KEY STAGE 3 – CONTENTS

(NOTE**: EACH** SECTION **BELOW CONTAINS AN** EXERCISE**, FOLLOWED BY** WORKED ANSWERS**)**

SECTION NO. **TOPIC**

9. **Average Speed**, **Distance** and **Time** – **problems** involving these (including worked examples) - **Travel graphs** (including worked examples)

10. **Shapes** – properties of **triangles** (scalene, isosceles & equilateral): **quadrilaterals** (square, rectangle, parallelogram, rhombus, kite, trapezium; areas, perimeters & symmetry are included): **pentagons**, **hexagons**, **heptagons** & **octagons**: **circles** (diameter, radius, circumference & area)): **length**, **area** and **volume** (including worked examples): **nets** (including worked examples): **symmetry** (line, plane, point, rotational, **order** of rotational symmetry)

11. **Angles** – **acute**, **obtuse** & **reflex** angles, **right** angle, **straight line** & **circle** – **drawing angles** with a **protractor** – **angle properties** of **straight lines** (**vertically opposite** angles, **corresponding angles**, **alternate angles** & **interior angles**)

12. **Introduction to algebra** – **addition**, **subtraction**, **multiplication** & **division** in **algebra**; **brackets**; **expanding** brackets; **powers**; **like terms** – **collating** like terms and **simplifying**; exercise (with worked answers); **practical applications** of **algebra** studied earlier; exercise (with worked answers)

13. **Equations** - **solving simple equations** – **construction** of (and **solving**) **simple equations** – worked examples

14. **Inequations** - **solving simple inequations** – **construction** of (and **solving**) **simple inequations** – **solving simple inequations** – showing **solution** to a **simple inequation** on a **number line** - worked examples

15. **Simultaneous equations** – **solving simultaneous equations** using **elimination** method – **solving simultaneous equations** containing **positive** and **negative values** of **x** and **y** – **solving simultaneous equations** by **graph**

16. **Statistics** – **mean**, **median**, **mode**, **tally chart**, **pie chart**, **bar graph**, **range**, (including worked examples)

17. **Probability** – **probability scale**

18. **Directions - cardinal points** & **three** – **figure bearings**

19. **Weights** and **measures - metric** / **imperial** conversions – **temperature scales** (**Fahrenheit** & **Centigrade**, including **conversion graph**)

20. **Simple** & **compound interest**; **appreciation** (increase in value)/ **depreciation** (decrease in value) - **formulae** given - worked examples

SECTION 1

ARITHMETICAL OPERATIONS - NUMBERS

I. ARITHMETICAL OPERATIONS

Definitions

1. The **sum** of 2 or more numbers is the **result** obtained by **adding** the numbers.

E.g. The **sum** of **2**, **3** and **5** is **2 + 3 + 5**
= **10**

The **order** of the numbers is **not important** in **addition**.

2. The **difference** between 2 numbers is the **result** obtained by **subtracting the smaller number from the larger number**.

E.g. The **difference** between **2** and **5** is **5 - 2**
= **3**

3. The **product** of 2 or more numbers is the **result** obtained by **multiplying** the numbers together.

E.g. (i) The **product** of **2** and **3** is **2 × 3**
= **6**

and

E.g. (ii) The **product** of **2, 3** and **5** is **2 × 3 × 5**
= **30**

The **order** of the numbers is **not important** in **multiplication**.

4. The **quotient** is the **result** obtained by **dividing one number by another**.

E.g. (i) The **quotient** $\mathbf{3 \div 2}$ is $\mathbf{\frac{3}{2}}$

but **E.g. (ii)** The **quotient** $\mathbf{2 \div 3}$ is $\mathbf{\frac{2}{3}}$ (i.e. they are **different**)

The **order** of the numbers is **important in division**.

Order of Operations

There are **4 basic operations** on numbers, namely :
Addition, **Subtraction**, **Multiplication** and **Division**.

Also, **brackets** are used to keep **numbers together**, to be treated as a **single quantity**. Think of brackets as a **pocket** which contains, let's say, a sum of money made up of several coins.

E.g. **£3.75** in a **pocket** is formed from **6** coins
($3 \times$ £1, $1 \times$ 50p, $1 \times$ 20p, $1 \times$ 5p)
i.e. **one** sum of money, but **six** coins.

When you are required to perform calculations involving **more than one of the operations** mentioned, a **certain order must be observed**.

The **order** is as follows:		**Mnemonic**:
(i)	Work out **Brackets**	**B**less
(ii)	**Multiply** and / or **Divide**	**M**y **D**ear
(iii)	**Add** and / or **Subtract**.	**A**unt **S**ally

NOTE : **Multiplication and Division** are done on the same line, as are **Addition and Subtraction**.

WORKED EXAMPLES USING ORDER OF OPERATIONS

Find the values of the following:

1. $5 + (3 - 2) \times 4 \div 2 - 3$

2. $2 + (7 - 6) \times 3$

3. $6 \times (4 - 2) + 1$

4. $7 - (6 \div 3) \times 4 + 2$

5. $6 - 8 \times 2 \div 4 + 3$

ANSWERS TO WORKED EXAMPLES USING ORDER OF OPERATIONS

1. $5 + (3 - 2) \times 4 \div 2 - 3$

$= 5 + 1 \times 4 \div 2 - 3$	**Brackets** removed first	**B**less
$= 5 + 2 - 3$	**Multiplication** and **Division** next	**M**y **D**ear
$= 4$	**Addition** and **Subtraction** last	**A**unt **S**ally

2. $2 + (7 - 6) \times 3$

$= 2 + 1 \times 3$	**Brackets** removed first	**B**less
$= 2 + 3$	**Multiplication** second	**M**y
$= 5$	**Addition** last	**A**unt

3. $6 \times (4 - 2) + 1$

$= 6 \times 2 + 1$	**Brackets** removed first	**B**less
$= 12 + 1$	**Multiplication** second	**M**y
$= 13$	**Addition** last	**A**unt

4. $7 - (6 \div 3) \times 4 + 2$

$= 7 - 2 \times 4 + 2$	**Brackets** removed first	**B**less
$= 7 - 8 + 2$	**Multiplication** second	**M**y
$= 1$	**Addition** and **Subtraction** last	**A**unt **S**ally

5. $6 - 8 \times 2 \div 4 + 3$

$= 6 - 4 + 3$	**Multiplication** and **Division** first	**M**y **D**ear
$= 5$	**Addition** and **Subtraction** last	**A**unt **S**ally

II. NUMBERS

1. PRIME NUMBERS = {2, 3, 5, 7, 11, 13, 17, 19, . . .}

A **prime number** has **no factors except itself and 1**.
This means that **no other number will divide into a prime number** without leaving a remainder.
E.g. 6 is **not** a prime number because it has **factors** of **3** and **2.**
On the other hand, **31 is a prime number** because no number except **31** and **1** will divide into it without leaving a remainder.

2. EVEN NUMBERS = {2, 4, 6, 8, 10, 12, 14, 16, . . .}

An **even number** is a number which **can be divided by 2** without leaving a remainder.
We could say it is a **multiple of 2** (see section 8).

3. ODD NUMBERS = {1, 3, 5, 7, 9, 11, 13, 15, . . .}

An **odd number** is a number which **when divided by 2 gives a remainder of 1**.
Eg. **21** is **odd** because when we **divide** it by **2**, we get a **remainder** of **1**.

4. SQUARE NUMBERS = {1, 4, 9, 16, 25, 36, 49, 64, . .}

Square numbers are called this because they give the **areas** of **squares** of **edge 1, 2, 3, . . .**

We have: $1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

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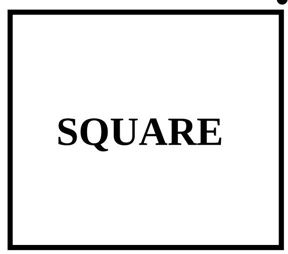

Square numbers are often referred to as **squares**.

5. **CUBIC NUMBERS** = $\{1, 8, 27, 64, 125, \ldots\}$

Cubic numbers give the **volumes of cubes of edge 1, 2, 3,**. . .

We have: $1^3 = 1 \times 1 \times 1 = 1$

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$

•
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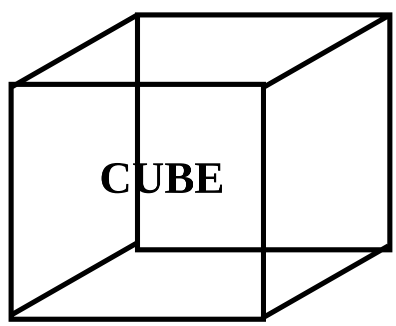
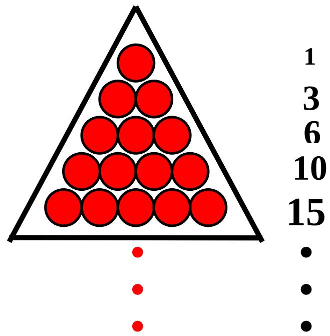


Cubic numbers are often referred to as **cubes.**

6. **TRIANGULAR NUMBERS** = $\{1, 3, 6, 10, 15, 21, \ldots\}$



Notice how this sequence progresses - the **difference between successive terms is increased by 1 each time**.
We have : **add 2, add 3, add 4, add 5, add 6, add 7** and so on.
This would remind you of the 15 red balls enclosed in a triangular frame before commencement of a snooker game.

7. **FIBONACCI NUMBERS =** $\{1, 1, 2, 3, 5, 8, 13, \ldots\}$

Add the last two numbers to get the one which follows.
It is easy to continue this sequence:
the **next term is 21, then 34** and so on.

8. **MULTIPLES** (Think of **Multiplication** tables)

(i) of 4 $= \{4, 8, 12, 16, 20, 24, \ldots\}$
(ii) of 5 $= \{5, 10, 15, 20, 25, 30, \ldots\}$
etc.

A **multiple** of any number, then, is a number into which **that number will divide**, without leaving a remainder.

9. **FACTORS divide exactly** into numbers, without leaving a remainder.

Eg. **1, 2, 3, 4, 6,** and **12** are **all the factors** of **12** and **1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30**, and **60** are **all the factors** of **60**.

10. **LOWEST COMMON MULTIPLE (L.C.M.)** of 9, 12 and 18 is **36**.
It is the **lowest number** into which **each** of **the numbers 9**, **12** and **18 will divide exactly**.

The **L.C.M.** of a **set of numbers** is easily found by considering the **multiplication tables** for the **larger (**or **largest)** of the numbers. The **first number in these multiplication tables** that can be divided exactly by **each of the other numbers** is the L.C.M.

Look again at the L.C.M. of 9, 12 and 18. Since **18 is the largest** of these, we consider the **18-times tables**:

$18 \times 1 = 18$ (can be divided exactly by 9, but **not** 12)
$18 \times 2 = \mathbf{36}$ (can be divided exactly by 9 **and** 12, so this is it!)

Therefore, **36** is the **L.C.M.** of **9, 12 and 18**.

11. **HIGHEST COMMON FACTOR (H.C.F.)** of 9, 12 and 18 is **3**.
It is the **highest** number that **can be divided** into **9**, **12** and **18**.

The H.C.F. of a set of numbers is easily found by considering the **prime factors** of each of the numbers. The **prime factor (**or **factors) common to each** of the numbers is the **H.C.F.**

Remember that a **prime factor** is a **prime number** that **divides exactly** into another number.
E.g. **2** is a **prime factor** of **6**,
since **2** is a **prime number** that **divides exactly** into **6**.

Prime Numbers = {2, 3, 5, . . .}

	9	$= 3 \times 3$	(in prime factors)
	12	$= 2 \times 2 \times 3$	(in prime factors)
and	18	$= 2 \times 3 \times 3$	(in prime factors)

There is a common factor of **3**, since there is **at least one 3** in each.
So the **H.C.F**. of **9, 12 and 18** is **3**.

We shall look at another example:

Find the **H.C.F.** of **12, 18** and **24**.

	12	$= 2 \times 2 \times 3$	(in prime factors)
	18	$= 2 \times 3 \times 3$	(in prime factors)
and	24	$= 2 \times 2 \times 2 \times 3$	(in prime factors)

This time we have **2 ´ 3 = 6** as the **H.C.F.**, since **2 ´ 3 is in each**.

EXERCISE 1
(Arithmetical Operations and Numbers)

1. Using the correct order for arithmetical operations, calculate the following:

(i) $3 - (2 \times 1) + 4 \div 8$

(ii) $(6 - 4) \times 3 + 10 \div 5$

(iii) $2 \times 3 - 2 \div 4 + 5$

(iv) $2 \times (3 \times 3) \div 3 + 1 - 2$

(v) $10 \div 5 + (6 - 4) \times 3$

(vi) $10 - 8 \div 2 + 7 \times 3$

(vii) $8 \div 4 + 4 \times 2 - 3$

2. Fill in the gaps in the following sequences:

(i) ___ , 3, ___ 7, 9, ___ , ___ , ___ , 17, . . .
(ii) ___ , 4, ___ , 8, 10, ___ , ___ , . . .
(iii) 2, ___ , 5, ___ , ___ , ___ , 17, 19, . . .
(iv) 1, ___ , 9, ___, ___ , 36, 49, . . .
(v) ___ , 8, 27, ___ , 125, . . .
(vi) 1, 1, ___ , 3, 5, ___ , 13, 21, . . .
(vii) 1, 3, ___ , 10, 15, ___ , ___ , . . .
(viii) ___ , 8, 12, ___ , 20, ___ , ___ , 32, . . .

3. Name each set of numbers in the sequences (i) to (viii) in question 2 above.

4. Write **all** the **factors** of 48.

5. Write **all** the **prime factors** of 48.

6. Find the **Highest Common Factor (H.C.F.)** of 24, 36 and 48.

7. Write down the first 6 **multiples** of 9.

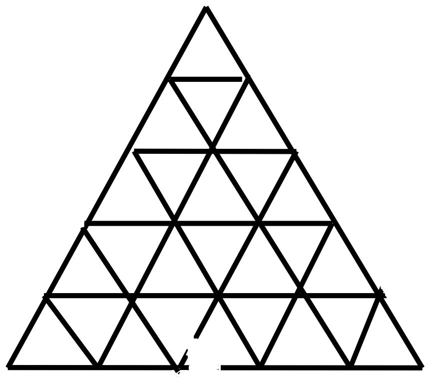
8. Find the **Lowest Common Multiple (L.C.M.)** of 10, 20 and 30.

9. A girl has 3 strips of ribbon measuring 12 cm, 20 cm and 24 cm. She wishes to cut the strips into smaller pieces of equal length so that there is no ribbon left over.
What is the longest that she can make each piece?

10. Outside a shop, there are flashing neon lights, coloured **yellow, green** and **red.**
The **yellow** light flashes every **3 seconds**,
the **green** light flashes every **4 seconds** and
the **red** light flashes every **5 seconds**.

If the lights **all flash together** as soon as they are switched on at **8.30 am**, what is the **earliest time** at which they will **all flash together again?**

11. (i) Complete the table below the diagram:
(Row 2 has been done)



. . . Row 1

. . . Row 2

. . . Row 3

. . . Row 4

. . . Row 5

.
.

Table

Row No.	**1**	**2**	**3**	**4**	**5**	**.**	**.**	**.**
No. of Triangles	_	3	_	_	_	**.**	**.**	**.**

(ii) Try to spot a quick method to find the number of triangles there would be in **Row 10**.

(iii) What is the quick method? Describe it in your own words.

(iv) Fill in the gaps below:
Total Number of Triangles in the first:

	1 row is	**1**
	2 rows is	**4**
	3 rows is	**9**
(a)	**4 rows** is	____
(b)	**5 rows** is	____
	•	
	•	
	•	
(c)	**9 rows** is	____
(d)	**10 rows** is	____

Now try and spot a quick way to work out the total number of triangles in **any number** of rows, without having to draw them on the diagram!

(v) What kind of numbers are these?
1, 4, 9, etc.

(vi) Using the quick methods, find:

(a) the **number** of **triangles** in **row 30**
and
(b) the **total number** of **triangles** in the **first 30 rows**.

EXERCISE 1 - ANSWERS

1. **(i)** $1\frac{1}{2}$ **(ii)** 8 **(iii)** $10\frac{1}{2}$ **(iv)** 5
   (v) 8 **(vi)** 27 **(vii)** 7

2. **(i)** 1, 5, 11, 13, 15 **(ii)** 2, 6, 12, 14 **(iii)** 3, 7, 11, 13
   (iv) 4, 16, 25 **(v)** 1, 64 **(vi)** 2, 8
   (vii) 6, 21, 28 **(viii)** 4, 16, 24, 28

3. **(i)** Odd numbers **(ii)** Even numbers (or multiples of 2)

   (iii) Prime numbers **(iv)** Square numbers **(v)** Cubic numbers

   (vi) Fibonacci Numbers **(vii)** Triangular numbers

   (viii) Multiples of 4

4. {1, 2, 3, 4, 6, 8, 12, 16, 24, 48} I hope you didn't forget 1 and 48 !

5. {2, 3} No other **prime number** divides exactly into 48.

6. 12

7. {9, 18, 27, 36, 45, 54}

8. 60

9. 4 cm. (You need the H.C.F. of 12 cm, 20 cm and 24 cm)

10. 8.31 a.m. (You need the L.C.M. of 3 sec, 4 sec and 5 sec.)

11. **(i)** 1, 5, 7, 9 **(ii)** 19 **(iii)** 2 times Row No. minus 1
    (iv)(a) 16 **(b)** 25 **(c)** 81 **(d)** 100 **(v)** Square numbers
    (vi)(a) 59 **(b)** 900

SECTION 2

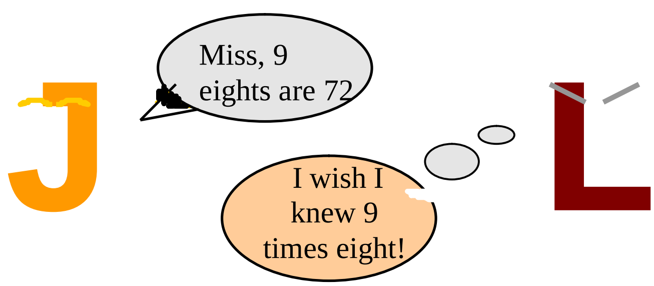
LONG MULTIPLICATION & DIVISION

Multiplication is a **short method of addition**.

E.g. **2 + 2 + 2 = 6**
can be found by **2 ´ 3**
since **2 is added 3 times over**.
Also **21 + 21 + 21 + 21 = 84**
can be found by **21 ´ 4**
since, this time, **21 is added 4 times over**.

For convenience, **MULTIPLICATION TABLES** have been formulated for the **2-times** up to the **12-times** and **all**, preferably, should be **learnt off by heart**. It is **essential,** however, that you **know all the tables** from the **2-times** to the **9-times**.

Long multiplication methods may be used for multiplication by the 2 digit numbers.



Cayley Multiplication Table

´	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication by a number **above 9** is easily performed if you know your tables up to and including the 9 times tables.

Multiplication by 10 is performed simply by adding **0** to the end of a number, **multiplication by 100** by adding **00**, **multiplication by 1000** by adding **000**, and so on.

Multiplication by 11 or by 12 may be performed by using **short multiplication methods**, if you know the **11 times -** and **12 times - tables**. Ideally you should.

However, if you do not know these tables, **long multiplication methods** may be used.

Long Multiplication

Remember your **place values** in numbers.
E.g.

		t	u
22	**=**	**2**	**2**

i.e. **22** is equivalent to **2 tens + 2**.
Then, **multiplication** of a number **by 22** is the same as **multiplying it by 20** and **adding on 2 times the number.**

Take 19×22

$$\begin{aligned} &= 19 \times 20 + 19 \times 2 \\ &= 380 + 38 \\ &= 418. \end{aligned}$$

We could do

$$\begin{aligned} &\ 22 \times 19 \\ &= 22 \times 10 + 22 \times 9 \\ &= 220 + 198 \\ &= 418. \end{aligned}$$

Notice that 19 ´ 22 = 22 ´ 19
just as **2 ´ 3 = 3 ´ 2.**

Long multiplication can be performed **all in one calculation**, **remembering place values before multiplying** at each stage.

Take **19 ´ 22** again

$$\begin{array}{rl} = \quad 19 & \\ \times\ 22 & \\ \hline 38 & \textbf{(i.e. 2 times 19)} \\ +\ 380 & \textbf{(i.e. 20 times 19)} \\ \hline 418 & \end{array} \qquad \textbf{or} \qquad \begin{array}{rl} 22 & \\ \times\ 19 & \\ \hline 198 & \textbf{(i.e. 9 times 22)} \\ +\ 220 & \textbf{(i.e. 10 times 22)} \\ \hline 418 & \end{array}$$

We can now proceed to multiplication by numbers with more than 2 digits, **again noting carefully place values** at each stage of the multiplication.

E.g.

$$\begin{aligned} & 342 \times 512 \\ = \; & 342 \times 500 + 342 \times 10 + 342 \times 2 \\ = \; & 171000 + 3420 + 684 \\ = \; & 175104. \end{aligned}$$

All in one calculation, we have:

$$\begin{array}{rl} 342 & \\ \underline{\times\, 512} & \\ 684 & \text{(i.e. 2 times 342)} \\ 3420 & \text{(i.e. 10 times 342)} \\ \underline{+\, 171000} & \text{(i.e. 500 times 342)} \\ \underline{175104}. & \end{array}$$

Long Division

Division is the **inverse process to multiplication**; a sort of **backwards multiplication**.

E.g.	$\mathbf{2 \times 3 = 6}$
gives	$\mathbf{6 \div 2 = 3}$
or	$\mathbf{6 \div 3 = 2}$.

Clearly, then, you **work from the multiplication tables** when doing division.

E.g. $\mathbf{18 \div 2}$

$$= \begin{array}{r} \mathbf{9} \\ \mathbf{2\overline{)18}} \end{array}$$

This says **2 into 18 goes 9 times**, because $\mathbf{2 \times 9 = 18}$.

There are different methods used for long division and in this book, I shall consider **2 methods**.

Method 1

Do out **multiplication tables of your own** and perform the division in **exactly the same way as for short division**.

E.g. $182 \div 13$

Then

$$13 \overline{)\ 182} \quad \text{quotient } \mathbf{014}$$

So $182 \div 13 =$ **14**

13-times tables

$13 \times 0 = 0$
+ 13
$13 \times 1 = 13$
+ 13
$13 \times 2 = 26$
+ 13
$13 \times 3 = 39$
+ 13
$13 \times 4 = 52$
•
•
•

Method 2

This is the usual method used for long division and it **involves 4 steps**, which are **repeated as often as required** until the division is complete.

The **4 steps** are:

(i) Divide
(ii) Multiply
(iii) Subtract
(iv) Bring down next figure

Then, **repeat steps (i) to (iv)** as many times as you find necessary.

$182 \div 13$

```
      014 Remainder 0
13 ) 182
    - 13ü
       52
     - 52
        0 = Remainder
```

The answer to **182 ¸ 13 is 14 times, remainder 0, i.e. 14 times exactly.**

4 Steps:

(i) 13 into 18 goes 1 time. (Put 1 in answer above 8).

(ii) Multiply 13 by 1 (in answer) = 13. Write this 13 under 18.

(iii) Subtract 13 from 18 above = 5.

(iv) Bring down next figure i.e. 2, and place it beside the 5. **(Tick figure when it is brought down).**

Repeat 4 steps:

(i) 13 into 52 goes 4 times. (Put 4 in answer above 2).

(ii) Multiply 13×4 (in answer) = 52.

(iii) Subtract 52 from 52 above = 0.

(iv) No next figure to bring down, so the division is complete.

WORKED EXAMPLES ON LONG DIVISION

(i) $2407 \div 17$

(ii) $3125 \div 21$

(iii) $2006 \div 31$

ANSWERS

Remember the **4 steps**:

(i) ÷

(ii) ×

(iii) -

(iv) Bring Down

(i) 2407 ÷ 17

```
    0141 Remainder 10
17 ) 2407
   - 17üü
      70
    - 68
      27
    - 17
      10  =  Remainder
```

The answer to **2407 ¸ 17** is **141 times, remainder 10**.

4 Steps.

(i) 17 into 24 goes 1 time. (Put 1 in answer above 4).
(ii) Multiply 17×1 (in answer) = 17. Write this 17 under 24.
(iii) Subtract 17 from 24 above = 7.
(iv) Bring down next figure i.e. 0 and place it beside the 7. (Tick figure when it is brought down).

Repeat 4 steps.

(i) 17 into 70 goes 4 times. (Put 4 in answer above 0).
(ii) Multiply 17×4 (in answer) = 68. Write 68 under 70.
(iii) Subtract 68 from 70 above = 2.
(iv) Bring down next figure i.e. 7 and place it beside the 2. (Tick figure when it is brought down).

Repeat 4 steps again.

(i) 17 into 27 goes 1 time. (Put 1 in answer above 7).
(ii) Multiply 17×1 (in answer) = 17. Write 17 under 27.
(iii) Subtract 17 from 27 above = 10.
(iv) No next figure to bring down, so the division is complete, and this time, there is a **remainder of 10**.

(ii)

```
       0148 Rem. 17
21 ) 3125
   - 21
     102
   - 84
      185
   - 168
       17
```

The answer to **3125 ¸ 21** is **148 times, remainder 17**.

(iii)

```
        0064 Rem. 22
31 ) 2006
   - 186
      146
    - 124
       22
```

The answer to **2006 ¸ 31** is **64 times, remainder 22**.

EXERCISE 2

(Long Multiplication & Division)

1.	2403 × 27	**2.**	1064 × 31	**3.**	2008 × 134
4.	1705 × 209	**5.**	3210 × 680	**6**.	200 ÷ 19
7.	1604 ÷ 14	**8.**	2003 ÷ 23	**9.**	1050 ÷ 29
10.	8004 ÷ 31.				

EXERCISE 2 - ANSWERS

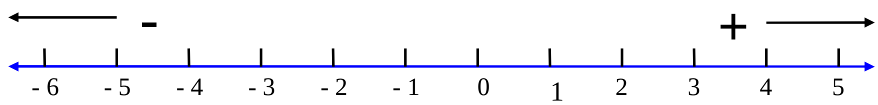
1. 64881
2. 32984
3. 269072
4. 356345
5. 2182800
6. 10 Rem. 10
7. 114 Rem. 8
8. 87 Rem. 2
9. 36 Rem. 6
10. 258 Rem. 6

SECTION 3

DIRECTED NUMBERS

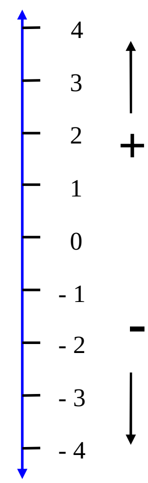
Number Line:

(i) Horizontal Line



OR

(ii) Vertical Line



Minus numbers are **less than 0** -
for example, temperatures often drop **below 0° Celsius** in Winter.

On the **horizontal** number line,
+ is the right direction and **- is the left direction**.

On the **vertical** number line,
+ is the upwards direction and **- is the downwards direction.**

Either the horizontal **or** the vertical number line may be used for **directed numbers**.

E.g. 1. **- 4 + 3 = -1,** i.e. go **3 moves in the + direction from - 4**.

E.g. 2 **- 5 + 7 = 2,** i.e. go **7 moves in the + direction from - 5**.

PRACTICAL EXAMPLES USING DIRECTED NUMBERS

Eg. 1 The temperature rises from - 6°C to 3°C.
Find the increase in temperature.

Method: Find **- 6** on the number line and count how many moves it takes to reach **3**.

You can see that it takes **6** to bring the temperature up to **0** and then another **3**, making an **increase** of **9** altogether.

Eg. 2 I owe my friend £25. If I pay him back £20, how much do I still owe?

Method: This is like **- 25 + 20 = -5,**

i.e. I still owe **£5.**

Eg. 3 I have a bank overdraft of £20. If I lodge £30 in the bank, how much money do I have in my account?

Method: A bank **overdraft** is like a **minus number** of **£** in the bank because I have **£20 less** than **£0**.

Then we have:

- 20 + 30 = 10.

Now I have cleared my overdraft and still have **£10** in my account.

EXERCISE 3

Using the **Number Line**, if necessary, answer the following questions:

1. The temperature changes from **- 5°C** to **- 1°C**.

(i) Is this an increase or a decrease?
(ii) How much is the increase or decrease?
(iii) Explain, in words, the reason for your answers to (i) and (ii) above.

2. **(a)** Try to work out the following, **without** using the Number Line:

(i) - 6 + 2
(ii) - 3 + 8
(iii) - 6 - 5
(iv) 5 + 6
(v) 7 - 8
(vi) 8 - 7
(vii) 3 - 8
(viii) 8 - 3
(ix) - 1 - 5
(x) 1 + 5

(b) Have you spotted a quick method to finding the answer when **adding plus and minus numbers**? Describe, in your own words, the quick method which you used.

3. In Omagh, on February 1st and February 2nd in a certain year, the following temperatures were recorded at the times shown:

DATE	TIME	TEMPERATURE
1st February	6.00 p.m.	3°C
	9.00 p.m.	1°C
	12.00 midnight	- 4°C
2nd February	3.00 a.m.	- 7°C
	6.00 a.m.	- 8°C
	9.00 a.m.	- 1°C
	12.00 noon	1°C
	3.00 p.m.	4°C
	6.00 p.m.	2°C

Using the information given in the table above, complete the following, making sure that you record your answer in the correct box each time, i.e. **Increase** or **Decrease** in **°C**.

DATE	**TIMES (from)**	**INCREASE °C**	**DECREASE °C**
1st February	**(i)** 6.00 p.m. to 9.00. p.m.	_____	_____
	(ii) 9.00 p.m. to 12.00 midnight	_____	_____
2nd February	**(iii)** 12.00 midnight to 3.00 a.m.	_____	_____
	(iv) 3.00 a.m. to 6.00 a.m.	_____	_____
	(v) 6.00 a.m. to 9.00 a.m.	_____	_____
	(vi) 9.00 a.m. to 12.00 noon	_____	_____
	(vii) 12.00 noon to 3.00 p.m.	_____	_____
	(viii) 3.00 p.m. to 6.00 p.m.	_____	_____

EXERCISE 3 - ANSWERS

1. **(i)** Increase

   (ii) 4°**C**

   (iii) The temperature changes from 5°**C** below 0° to just 1°C below 0° - therefore it is 4°**C** higher.

2. **(a)(i)** - 4

   (ii) 5

   (iii) - 11

   (iv) 11

   (v) - 1

   (vi) 1

   (vii) - 5

   (viii) 5

   (ix) - 6

   (x) 6

   (b) Yes. Subtract one from the other, then the answer has the **same sign as the larger number**.

3. **(i)** 2°C Decrease

   (ii) 5°C Decrease

   (iii) 3°C Decrease

   (iv) 1°C Decrease

   (v) 7°C Increase

   (vi) 2°C Increase

   (vii) 3°C Increase

   (viii) 2°C Decrease

SECTION 4

FRACTIONS

What is a fraction?

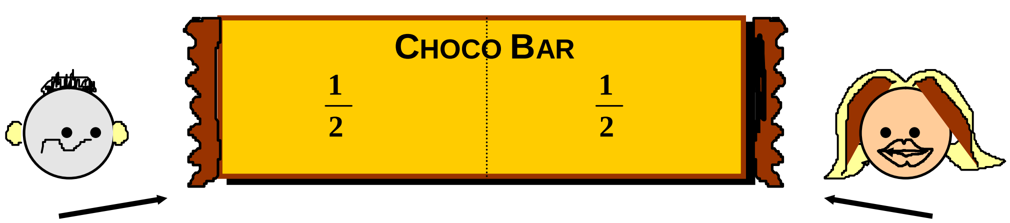
A **fraction** is a **part** of something, that is, a **bit broken off the whole**.

Let us pretend that you have a Choco Bar, which you are about to eat, when unexpectedly, your friend calls in. What do you do? Well, being a decent person, you would want to share the bar equally with your friend!

Now you do **not** have a **whole** bar for yourself, but you have a **half-bar each**.

A **half** is written as $\frac{1}{2}$ and since **2 halves make one whole**, we have:

$$2 \times \frac{1}{2} \quad \text{or} \quad \frac{2}{2} \quad = \mathbf{1\ whole}.$$



Just as you and your friend are about to divide up the Choco Bar, 2 more friends drop in!
Oh! Dear! What do you do? Again, you would want to share the bar with your 3 friends, wouldn't you?

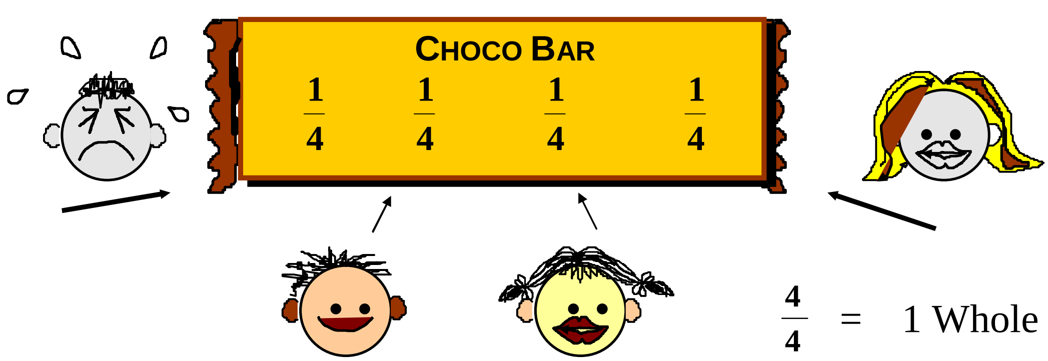
Now that the bar must be broken up into 4 equal parts, you get just **one quarter** of a bar each.

A **quarter** is written as $\frac{1}{4}$ and since **4 quarters make one whole**, we have:

$$4 \times \frac{1}{4} \quad \text{or} \quad \frac{4}{4} = 1 \text{ whole}.$$

Notice that **2 quarters** is the same as **1 half**, so :

$$\frac{2}{4} = \frac{1}{2}$$



$$\frac{4}{4} = 1 \text{ Whole}$$

$$\frac{2}{4} = \frac{1}{2}$$

EQUIVALENT FRACTIONS

Since $\frac{2}{4}$ and $\frac{1}{2}$ are the **same**, they are called **equivalent fractions**. Also $\frac{4}{8}$ and $\frac{1}{2}$ are equivalent, as are $\frac{4}{6}$ and $\frac{2}{3}$, and so on.

LOWEST TERMS

$\mathbf{\frac{1}{2}}$ **is in its lowest terms,** but $\mathbf{\frac{2}{4}}$ is **not** in its **lowest terms** because the **top** and **bottom** can **both** be **divided** by **2**.

The **top part** of a fraction is called **the numerator** and the **bottom part** is called **the denominator**.

If **both** the numerator and the denominator **can be divided exactly by the same number**, the fraction can **be reduced to its lowest terms**.

WORKED EXAMPLES

Reduce the following to their lowest terms, where necessary :-

(i) $\frac{3}{4}$ **(ii)** $\frac{5}{10}$ **(iii)** $\frac{8}{12}$

ANSWERS

(i) $\mathbf{\frac{3}{4}}$ **is** already in its lowest terms, since there is no number that will divide exactly into 3 and 4.

(ii) $\mathbf{\frac{5}{10}}$ **is not** in its lowest terms, since **top and bottom can both be divided by 5**.

We have: $\mathbf{\frac{5}{10}} = \mathbf{\frac{1}{2}}$ in its lowest terms.

(iii) $\frac{\mathbf{8}}{\mathbf{12}}$ is **not** in its lowest terms since **top** and **bottom** can **both** be **divided** by **4**.

We have:

$\frac{\mathbf{8}}{\mathbf{12}} = \frac{\mathbf{2}}{\mathbf{3}}$ in its lowest terms.

PROPER FRACTIONS, IMPROPER FRACTIONS & MIXED NUMBERS

A **proper fraction** is **less** than **one whole**, that is, the **numerator** is **smaller** than the **denominator**.

E.g. $\frac{\mathbf{3}}{\mathbf{4}}$ is a **proper fraction**.

An **improper fraction** is **more** than **1 whole**, that is, the **numerator** is **larger** than the **denominator**.

E.g. $\frac{\mathbf{7}}{\mathbf{4}}$ is an **improper fraction**.

Sometimes these are called '**top-heavy'** fractions.

A **mixed number** is **partly whole** and **partly fraction**.

E.g. $\mathbf{1}\frac{\mathbf{3}}{\mathbf{4}}$ is a **mixed number**.

Notice that the **improper fraction** $\frac{\mathbf{7}}{\mathbf{4}}$ is equal to the **mixed number** $\mathbf{1}\frac{\mathbf{3}}{\mathbf{4}}$.

Look again at the Choco Bar.

$\frac{\mathbf{7}}{\mathbf{4}}$ is 1 whole bar plus $\frac{\mathbf{3}}{\mathbf{4}}$ of another bar.

Improper Fractions are easily **changed** to **mixed numbers** by **dividing top** by **bottom**, thus:

$$\frac{\mathbf{7}}{\mathbf{4}} = 7 \div 4 = \mathbf{1}\frac{\mathbf{3}}{\mathbf{4}}.$$

Mixed numbers are **changed** into **improper fractions** by **multiplying** the **whole part** by the **denominator** and **adding on** the **numerator** to get the **numerator** for the **improper fraction** - the **denominator remains the same**.

Taking $1\frac{3}{4}$**, we have**:

1 ´ 4 + 3 over 4 = $\frac{7}{4}$ as **an improper fraction**.

ADDITION & SUBTRACTION OF FRACTIONS

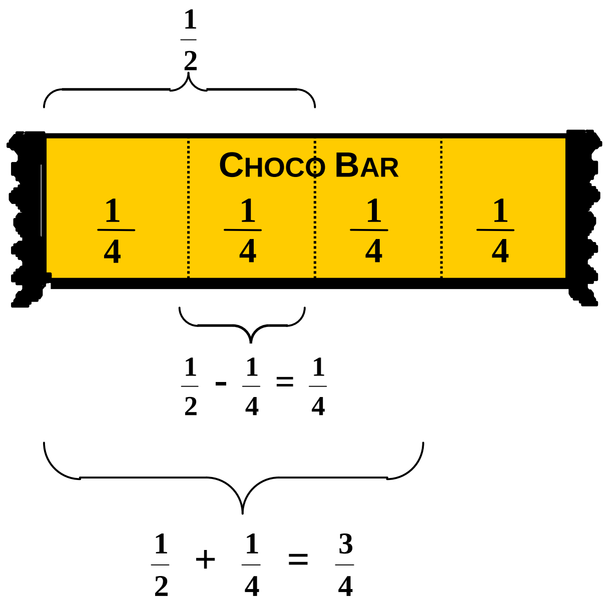
Look again at the Choco Bar.

If we wish to **add** $\frac{1}{2} + \frac{1}{4}$, we must **change the** $\frac{1}{2}$ into $\frac{2}{4}$ **before** we start.

Then: $\frac{1}{2} + \frac{1}{4}$

$= \frac{2}{4} + \frac{1}{4}$

$= \frac{3}{4}$



Also, if we wish to **subtract** $\frac{1}{4}$ from $\frac{1}{2}$, we must **change** the $\frac{1}{2}$ into $\frac{2}{4}$ first.

Then: $\frac{1}{2} - \frac{1}{4}$

$= \frac{2}{4} - \frac{1}{4}$

$= \frac{1}{4}$

Clearly, then, **we must have the same denominator in fractions before we start to add or subtract.**

WORKED EXAMPLES

1. $\frac{1}{2} + \frac{1}{6}$ **2.** $\frac{5}{8} - \frac{1}{2}$ **3.** $\frac{1}{2} + \frac{1}{3}$

4. $\frac{2}{5} + \frac{3}{10}$ **5.** $\frac{2}{3} - \frac{1}{2}$ **6.** $1\frac{2}{5} + \frac{1}{3}$

7. $2\frac{1}{4} - 1\frac{3}{5}$

ANSWERS TO WORKED EXAMPLES

1. $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \mathbf{\frac{2}{3}}$ in its lowest terms.

2. $\frac{5}{8} - \frac{1}{2} = \frac{5}{8} - \frac{4}{8} = \mathbf{\frac{1}{8}}$.

3. $\frac{1}{2} + \frac{1}{3}$

Multiplying the **2** by **3**, we get **6** as a **common denominator**.

Then: $\frac{1}{2} = \frac{3}{6}$ and : $\frac{1}{3} = \frac{2}{6}$ gives : $\frac{3}{6} + \frac{2}{6} = \mathbf{\frac{5}{6}}$.

4. $\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \mathbf{\frac{7}{10}}$.

5. $\frac{2}{3} - \frac{1}{2}$

Multiplying the **3** by **2**, we get **6** as a **common denominator**.

Then: $\frac{2}{3} = \frac{4}{6}$ and : $\frac{1}{2} = \frac{3}{6}$ gives : $\frac{4}{6} - \frac{3}{6} = \mathbf{\frac{1}{6}}$.

6. $1\frac{2}{5} + \frac{1}{3}$

Changing $1\frac{2}{5}$ to improper fraction form first, we have:

$$\frac{7}{5} + \frac{1}{3}$$

Multiplying the **5** by **3**, we get **15** as a **common denominator**.

Then: $\frac{7}{5} = \frac{21}{15}$ and : $\frac{1}{3} = \frac{5}{15}$ gives : $\frac{21}{15} - \frac{5}{15} = \frac{16}{15} = \mathbf{1\frac{1}{15}}$.

7. $2\frac{1}{4} - 1\frac{3}{5}$

Changing these mixed numbers into improper fractions, we have :

$$\frac{9}{4} - \frac{8}{5}$$

Multiplying the **4** by **5**, we get **20** as a **common denominator**.

Then: $\frac{9}{4} = \frac{45}{20}$ and: $\frac{8}{5} = \frac{32}{20}$ gives : $\frac{45}{20} - \frac{32}{20} = \mathbf{\frac{13}{20}}$.

MULTIPLICATION OF FRACTIONS

To **multiply fractions,** simply **multiply** the **numerators** together, and **multiply** the **denominators** together, to get the answer, which you should give in its **lowest terms**.

Also, if the **answer turns out** to be an **improper fraction**, it is usual to turn it into a **mixed number**.

N.B. Before **starting to multiply** fractions**, change any mixed numbers** into **improper fractions** first - this is really important.

WORKED EXAMPLES

(i) $\frac{2}{3} \times \frac{5}{7}$ **(ii)** $\frac{3}{4} \times \frac{2}{5}$ **(iii)** $1\frac{1}{2} \times \frac{4}{5}$

(iv) $2\frac{1}{4} \times 1\frac{2}{3}$ **(v)** $\frac{2}{5} \times 1\frac{3}{4}$

ANSWERS

(i) $\frac{2}{3} \times \frac{5}{7} = \frac{2\times 5}{3\times 7} = \mathbf{\frac{10}{21}}$

(ii) $\frac{3}{4} \times \frac{2}{5} = \frac{3\times 2}{4\times 5} = \frac{6}{20} = \mathbf{\frac{3}{10}}$ in its lowest terms.

(iii) $1\frac{1}{2} \times \frac{4}{5} = \frac{3}{2} \times \frac{4}{5} = \frac{3\times 4}{2\times 5} = \frac{12}{10} = 1\frac{2}{10}$

$= \mathbf{1\frac{1}{5}}$ in its lowest terms.

(iv) $2\frac{1}{4} \times 1\frac{2}{3} = \frac{9}{4} \times \frac{5}{3} = \frac{9\times 5}{4\times 3} = \frac{45}{12} = 3\frac{9}{12}$

$= \mathbf{3\frac{3}{4}}$ in lowest terms.

(v) $\frac{2}{5} \times 1\frac{3}{4} = \frac{2}{5} \times \frac{7}{4} = \frac{2\times 7}{5\times 4} = \frac{14}{20}$

$= \mathbf{\frac{7}{10}}$ in lowest terms.

DIVISION BY A FRACTION

Before starting division, **change any mixed numbers** into **improper fractions**. Then**, turn the divisor upside-down and multiply**. So **division** is really **upside - down multiplication**.

WORKED EXAMPLES

(i) $\frac{1}{2} \div \frac{1}{4}$

(ii) $\frac{2}{3} \div \frac{2}{5}$

(iii) $1\frac{1}{4} \div 2\frac{1}{4}$

(iv) $2\frac{1}{3} \div \frac{3}{5}$

(v) $\frac{5}{8} \div 1\frac{1}{3}$

ANSWERS

(i) $\frac{1}{2} \div \frac{1}{4}$

divisor

$= \frac{1}{2} \times \frac{4}{1}$ (Note : **only divisor** turned upside down)

$= \frac{1 \times 4}{2 \times 1} = \frac{4}{2} =$ **2**.

(ii) $\frac{2}{3} \div \frac{2}{5}$

divisor

$= \frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2} = \frac{10}{6} = \frac{5}{3} = \mathbf{1\frac{2}{3}}$.

(iii) $1\frac{1}{4} \div 2\frac{1}{4}$

$= \frac{5}{4} \div \frac{9}{4}$

$= \frac{5}{4} \times \frac{4}{9} \quad = \frac{5\times 4}{4\times 9} \quad = \frac{20}{36} = \mathbf{\frac{5}{9}}$

(iv) $2\frac{1}{3} \div \frac{3}{5} \quad = \frac{7}{3} \div \frac{3}{5} \quad = \frac{7}{3} \times \frac{5}{3}$

$= \frac{7\times 5}{3\times 3} \quad = \frac{35}{9} \quad = \mathbf{3\frac{8}{9}}$

(v) $\frac{5}{8} \div 1\frac{1}{3} \quad = \frac{5}{8} \div \frac{4}{3}$

$= \frac{5}{8} \times \frac{3}{4} \quad = \frac{5\times 3}{8\times 4} \quad = \mathbf{\frac{15}{32}}$

FINDING A FRACTION OF A QUANTITY

To **find a fraction** of a quantity, **divide by the denominator** and **multiply by the numerator**.

E.g. Find $\frac{3}{4}$ of 12 boys

$$\begin{array}{rl} 4\,\overline{)\,12} & \\ 3 & \longleftarrow \; = \frac{1}{4} \text{ of } 12 \\ \times\ 3 & \\ \hline 9 & \longleftarrow \; = \frac{3}{4} \text{ of } 12 \end{array}$$

So, $\frac{3}{4}$ of 12 boys is **9 boys**.

WRITING ONE NUMBER AS A FRACTION OF ANOTHER

To **write one number** as a **fraction** of **another**, simply **write one over** the **other** and give the fraction in its lowest terms.

E.g. Write **9 boys** as a fraction of **12 boys**.

$\mathbf{\frac{9}{12}} = \mathbf{\frac{3}{4}}$, in its lowest terms.

So, 9 boys out of 12 boys is $\mathbf{\frac{3}{4}}$.

WORKED PRACTICAL EXAMPLES ON FRACTIONS

QUESTIONS

1. A boy spends $\frac{1}{4}$ of his money on a book and $\frac{1}{3}$ of the remainder on a present. If he has £4.40 left how much had he to begin with?

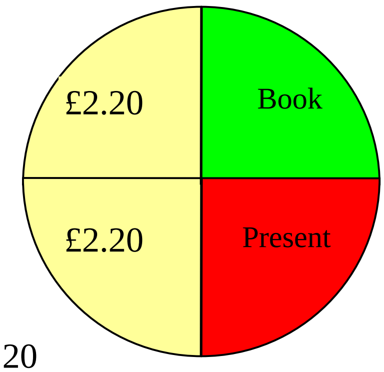
2. A girl gives $\frac{2}{5}$ of her pocket money to her little brother and $\frac{1}{2}$ of her pocket money to her little sister. If she has 85p left, how much had she to begin with?

3. If $\frac{2}{3}$ of a boy's marbles is 24 marbles, how many marbles has he altogether?

ANSWERS

Note: It is always helpful to draw a diagram e.g. a Pie Chart, when dealing with fraction problems of this type.

1.



$$\begin{array}{r} £2.20 \\ \underline{\times \quad 4} \\ \underline{\mathbf{£8.80}} \end{array}$$ **altogether**

Method:

1. Draw a Pie Chart.
2. Divide it into quarters.
3. Mark off one quarter for the book.
4. Then, clearly, one third of the remainder is another quarter of the pie chart which goes for the present.
5. Two quarters are left and they are equal to £4.40.
6. One quarter equals £2.20.
7. Four quarters equal £8.80.

2. The total fraction of her money given away is:

$$\frac{2}{5} + \frac{1}{2}$$

$$= \frac{4}{10} + \frac{5}{10}$$

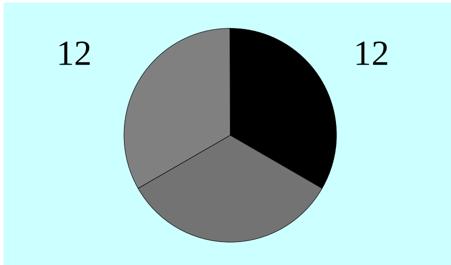
$$= \mathbf{\frac{9}{10}} \text{ of money gone.}$$

Therefore, she had $\mathbf{\frac{1}{10}}$ of her money **left**.

If $\mathbf{\frac{1}{10} = 85p}$

$\mathbf{\frac{10}{10} = £8.50}$.

3.



12

$$\frac{2}{3} = 24$$

Therefore, $\frac{1}{3} = 12$

and $\frac{3}{3} = 12 \times 3$

36 marbles.

EXERCISE 4
(Fractions)

1. The following fractions are equivalent (i.e. the same).
Fill in the correct figure in each box:

(i) $\frac{1}{2} = \frac{\square}{6}$ **(ii)** $\frac{1}{3} = \frac{2}{\square}$

(iii) $\frac{\square}{5} = \frac{6}{15}$ **(iv)** $\frac{8}{13} = \frac{\square}{26}$

(v) $\frac{5}{8} = \frac{20}{\square}$

2.

(i) $\frac{1}{4} + \frac{1}{3}$ **(ii)** $\frac{1}{2} + \frac{1}{5}$

(iii) $\frac{2}{3} + \frac{1}{5}$ **(iv)** $\frac{1}{5} + \frac{2}{7}$

(v) $\frac{2}{5} + \frac{3}{7}$

3.

(i) $\frac{1}{3} - \frac{1}{4}$ **(ii)** $\frac{1}{2} - \frac{1}{3}$

(iii) $\frac{2}{3} - \frac{1}{5}$ **(iv)** $\frac{4}{5} - \frac{1}{2}$

(v) $\frac{9}{10} - \frac{4}{5}$

4. Change the following **mixed numbers** into **improper fractions:** Eg. $1\frac{1}{2} = \frac{3}{2}$

mixed number improper fraction

(i) $1\frac{1}{3}$ **(ii)** $2\frac{1}{4}$ **(iii)** $1\frac{2}{5}$

(iv) $2\frac{4}{7}$ **(v)** $3\frac{5}{8}$

5. Change the following **improper fractions** into **mixed numbers** :-

(i) $\frac{5}{4}$ **(ii)** $\frac{8}{3}$ **(iii)** $\frac{7}{6}$

(iv) $\frac{10}{7}$ **(v)** $\frac{9}{5}$

6. Write each of the following sets of fractions in **ascending** order of size.

E.g. $\mathbf{\frac{2}{3}}$ $\mathbf{\frac{3}{4}}$ $\mathbf{\frac{1}{2}}$

Find a common denominator first:

12 is the common denominator

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

Therefore, $\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4}$ is the correct order.

Do the same for each of the following:

(i) $\frac{4}{5} \quad \frac{2}{3} \quad \frac{11}{15}$ **(ii)** $\frac{5}{8} \quad \frac{1}{2} \quad \frac{4}{5}$

(iii) $\frac{3}{8} \quad \frac{1}{3} \quad \frac{2}{5}$ **(iv)** $\frac{2}{3} \quad \frac{3}{4} \quad \frac{2}{5}$

(v) $\frac{7}{10} \quad \frac{3}{5} \quad \frac{1}{2}$

7. Find:

(i) $\frac{3}{4}$ of £20.00. **(ii)** $\frac{5}{8}$ of 24 people.

(iii) $\frac{6}{7}$ of 14 boys. **(iv)** $\frac{7}{9}$ of 27 girls.

(v) $\frac{2}{5}$ of 10 marbles.

8. Write each of the following as a fraction in its lowest terms:

(i) 2 cats as a fraction of 6 cats.
(ii) £12 as a fraction of £15.
(iii) 8 sweets as a fraction of 12 sweets.
(iv) 6 ponies as a fraction of 9 ponies.
(v) There are 6 black dogs and 12 white dogs in a pet show. What fraction of the dogs are:

(a) black
(b) white?

9.

(i) $\frac{1}{4} \times \frac{2}{3}$ **(ii)** $\frac{2}{5} \times \frac{1}{3}$ **(iii)** $\frac{4}{5} \times \frac{1}{2}$

(iv) $1\frac{3}{8} \times \frac{3}{8}$ **(v)** $1\frac{1}{4} \times 2\frac{1}{2}$

10.

(i) $\frac{1}{4} \div \frac{1}{3}$ **(ii)** $\frac{3}{8} \div \frac{4}{5}$ **(iii)** $\frac{3}{4} \div \frac{1}{2}$

(iv) $1\frac{1}{2} \div \frac{1}{3}$ **(v)** $1\frac{1}{4} \div 1\frac{1}{2}$.

EXERCISE 4 - ANSWERS

1	**(i)**	3	**(ii)**	6	**(iii)**	2	**(iv)**	16	**(v)**	32
2.	**(i)**	$\frac{7}{12}$	**(ii)**	$\frac{7}{10}$	**(iii)**	$\frac{13}{15}$	**(iv)**	$\frac{17}{35}$	**(v)**	$\frac{29}{35}$
3.	**(i)**	$\frac{1}{12}$	**(ii)**	$\frac{1}{6}$	**(iii)**	$\frac{7}{15}$	**(iv)**	$\frac{3}{10}$	**(v)**	$\frac{1}{10}$
4.	**(i)**	$\frac{4}{3}$	**(ii)**	$\frac{9}{4}$	**(iii)**	$\frac{7}{5}$	**(iv)**	$\frac{18}{7}$	**(v)**	$\frac{29}{8}$
5.	**(i)**	$1\frac{1}{4}$	**(ii)**	$2\frac{2}{3}$	**(iii)**	$1\frac{1}{6}$	**(iv)**	$1\frac{3}{7}$	**(v)**	$1\frac{4}{5}$

6. **(i)** $\boldsymbol{\frac{2}{3}}$ $\frac{11}{15}$ $\frac{4}{5}$ **(ii)** $\boldsymbol{\frac{1}{2}}$ $\frac{5}{8}$ $\frac{4}{5}$ **(iii)** $\boldsymbol{\frac{1}{3}}$ $\frac{3}{8}$ $\frac{2}{5}$

(iv) $\frac{2}{5}$ $\frac{2}{3}$ $\frac{3}{4}$ **(v)** $\frac{1}{2}$ $\frac{3}{5}$ $\frac{7}{10}$

7. **(i)** £15.00 **(ii)** 15 people **(iii)** 12 boys

(iv) 21 girls **(v)** 4 marbles.

8. **(i)** $\frac{1}{3}$ **(ii)** $\frac{4}{5}$ **(iii)** $\frac{2}{3}$ **(iv)** $\frac{2}{3}$

(v)(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$

9. **(i)** $\frac{1}{6}$ **(ii)** $\frac{2}{15}$ **(iii)** $\frac{2}{5}$

(iv) $\frac{33}{64}$ **(v)** $3\frac{1}{8}$

10. **(i)** $\frac{3}{4}$ **(ii)** $\frac{15}{32}$ **(iii)** $1\frac{1}{2}$

(iv) $4\frac{1}{2}$ **(v)** $\frac{5}{6}$

SECTION 5

DECIMAL NUMBERS (Latin : *decem* means 'ten')

Place Values:

Th.	H.	T.	U.	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
2	6	1	5	.	3	6	7	9	8

Notice that the **decimal point follows the units** and the **first decimal place is tenths**, the **second decimal place** is **hundredths**, the **third decimal place** is **thousandths**, etc.

E.g. (i) $0.3 = \frac{3}{10}$

E.g. (ii) $0.06 = \frac{6}{100}$

E.g. (iii) $0.007 = \frac{7}{1000}$

E.g. (iv) $0.0009 = \frac{9}{10000}$

E.g. (v) $0.00008 = \frac{8}{100000}$

A **decimal** number, then, is clearly **less than 1 whole**, and is simply a **fraction whose denominator is 10, 100, 1000, etc**.
You can see that the **number of decimal places** is **the same** as the **number of 0s** in the denominator. This is a useful '**rule of thumb**' when **changing decimals** to **fractions**.

1. Changing Decimals to Fractions

(i) $0.5 = \frac{5}{10} = \frac{1}{2}$ (**1 decimal place, 1 nought** in denominator)

(ii) $0.02 = \frac{2}{100} = \frac{1}{50}$ (**2 decimal places, 2 noughts** in denominator)

(iii) $0.25 = \frac{25}{100} = \frac{1}{4}$ (**2 decimal places, 2 noughts** in denominator)

(iv) $0.125 = \frac{125}{1000} = \frac{1}{8}$ (**3 decimal places, 3 noughts** in denominator)

(v) $0.75 = \frac{75}{100} = \frac{3}{4}$ (**2 decimal places, 2 noughts** in denominator)

(vi) $0.012 = \frac{12}{1000} = \frac{3}{250}$ (**3 decimal places, 3 noughts** in denominator)

(vii) $1.06 = 1\frac{6}{100} = 1\frac{3}{50}$ (**1 whole, plus decimal part** here)

(viii) $20.005 = 20\frac{5}{1000} = 20\frac{1}{200}$ (**20 wholes, plus decimal part** here)

2. Changing Fractions into Decimals

A fraction is $\frac{\textbf{numerator}}{\textbf{denominator}}$ which **means numerator ¸ denominator**.

Eg. (i) $\frac{1}{5} = 1 \div 5$

$= 5 \enspace \underline{)\,1.0}$

$\quad 0.2$ Answer . . . $\frac{\mathbf{1}}{\mathbf{5}} = 0.2$

Eg. (ii) $\frac{1}{4} = 1 \div 4$

$= 4 \enspace \underline{)\,1.00}$

$\quad 0.25$ Answer . . . $\frac{\mathbf{1}}{\mathbf{4}} = 0.25$

Eg. (iii) $\frac{\mathbf{3}}{\mathbf{4}} = 3 \div 4$

$= 4 \enspace \underline{)\,3.00}$

$\quad 0.75$ Answer . . . $\frac{\mathbf{3}}{\mathbf{4}} = 0.75.$

Eg. **(iv)** $\frac{\mathbf{5}}{\mathbf{8}} = 5 \div 8$

$= 8 \;\underline{)\,5.000}$

$= \quad 0.625$ Answer . . . $\frac{\mathbf{5}}{\mathbf{8}} = 0.625.$

To **change a fraction** into a **decimal**, then**,** take the **following steps**:

Step 1 Write down the numerator.
Step 2 Insert a decimal point, followed by **one or more 0s**.
Step 3 Divide by the **denominator**, remembering to keep the **decimal points lined up**.

Addition and Subtraction of Decimals

- keep decimal points in a vertical line to **preserve place values**.

Eg. (i) 0.2 + 1.035

$$\begin{array}{rr} = & 0.200 \\ + & \underline{1.035} \\ & \underline{1.235} \end{array}$$

Eg. (ii) 6 - 0.25

$$\begin{array}{rr} = & 6.00 \\ - & \underline{0.25} \\ & \underline{5.75} \end{array}$$

4. Multiplication of Decimals

Eg. (i) $0.2 \times 0.1 = 0.02$

Eg. (ii) $1.25 \times 0.03 = 0.0375$

When you are multiplying 2 decimal numbers, **first multiply** the **numbers** together, **ignoring the decimal points** and then **fix the decimal point in the answer**, taking into account **the total number of decimal places in the 2 numbers**.

5. Division of Decimals

The **divisor** must **not** have a **decimal point**.
If the divisor has a point, **it must be moved** before the division takes place.

E.g. (i) **1.25 ¸ 0.5**
First, write it like this:

0.5) 1.25

Clearly, we are required to **move the decimal point one place to the right to make the divisor a whole number**. To compensate for this change in the divisor, we **must do the same to the number which is being divided**, i.e**. move the decimal point one place to the right** in it also.

This gives:

5) 12.5
2.5 = Answer

E.g. (**ii**) 1.6 ÷ 0.02
= 0.02) 1.6

Here, we must **move the decimal point two places to the right** to make the **divisor whole**. This gives:

2) 160
80 = Answer

N.B. In example (**ii**) above, the decimal point disappears out of both numbers, thus giving a whole number answer.

WORKED EXAMPLES ON DECIMALS

1. Add : 1.2, 0.04 and 13.

2. Subtract 0.125 from 3.4.

3. Multiply 2.7 by 0.02.

4. Divide 12.054 by 0.02.

ANSWERS

1.

$$\begin{array}{r} 1.20 \\ 0.04 \\ + \ \underline{13.00} \\ \mathbf{14.24} \end{array} \ = \ \textbf{Answer}$$

2.

$$\begin{array}{r} 3.400 \\ - \ \underline{0.125} \\ \mathbf{3.275} \end{array} \ = \ \textbf{Answer}$$

3.

$$\begin{array}{l} \quad 2.7 \\ \times \ \underline{0.02} \\ \quad \mathbf{0.054} \end{array} \ = \ \textbf{Answer}$$

4.

$$0.02 \ \overline{)\ 12.054}$$

$$= \quad 2 \ \overline{)\ 1205.4}$$

$$\mathbf{602.7} \ = \ \textbf{Answer}$$

EXERCISE 5

1. Find the sum of the following:
1.07, 12.1 and 176. (Definition: '**Sum**' $\Rightarrow$ '**Add**')

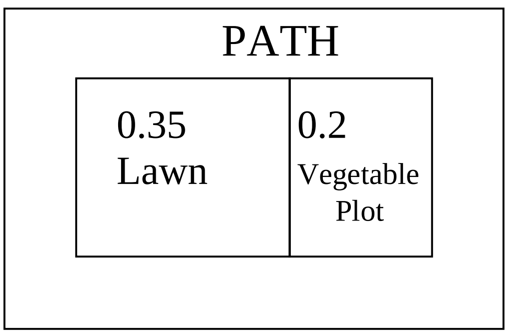
2. Find the difference between:
0.07 and 7. (Definition: '**Difference**' $\Rightarrow$ '**Subtract**')

3. Find the product of:
1.2 and 0.02. (Definition: '**Product**' $\Rightarrow$ '**Multiply**')

4. How many times can 0.6 be divided into 4.2?

5. Find the difference between 0.1 and $(0.1)^2$.

6.



In a garden, 0.35 of the total area is lawn, 0.2 is a vegetable plot, and the remainder is a path.

(i) Find in decimal form, the amount occupied by the path.

(ii) Write as a fraction, in its lowest terms, the part of the garden given over to:

(a) The lawn

(b) The vegetable plot

(c) The path.

7. 8.9 is $8\frac{9}{10}$ and is close to **9 wholes**.

7.15 is $7\frac{15}{100}$ and is close to **7 wholes**.

We say that **8.9** is **'rounded off'** to **9 wholes** and that **7.15** is **'rounded off'** to **7 wholes**. In the same way, write the following numbers '**rounded off**' to the **nearest whole number**:

(i)	0.8	**(ii)**	1.09	**(iii)**	17.78	**(iv)**	0.01
(v)	61.18	**(vi)**	16.75	**(vii)**	5.06	**(viii)**	112.7
(ix)	0.08	**(x)**	10.08				

8. Change the following decimal numbers into **fractions** in their **lowest terms**:

(i)	0.2	**(ii)**	0.07	**(iii)**	0.25	**(iv)**	0.75
(v)	0.95	**(vi)**	0.06	**(vii)**	0.35	**(viii)**	0.9
(ix)	0.003	**(x)**	0.015				

9. Change the following fractions into **decimals**:

(i)	$\frac{1}{4}$	**(ii)**	$\frac{3}{4}$	**(iii)**	$\frac{4}{5}$	**(iv)**	$\frac{1}{8}$
(v)	$\frac{2}{5}$	**(vi)**	$\frac{3}{8}$	**(vii)**	$\frac{1}{5}$	**(viii)**	$\frac{1}{2}$
(ix)	$\frac{7}{8}$	**(x)**	$\frac{3}{5}$				

10. Fill in the blank spaces:

(i) 0.1 + 0.15 + 0.09 + ______ = 1 whole.
(ii) 0.6 + 0.05 - ______ = 1 - 0.45.

EXERCISE 5 - ANSWERS

1. 189.17

2. 6.93

3. 0.024

4. 7 times.

5. 0.09.

6. **(i)** 0.45 **(ii) (a)** $\frac{7}{20}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{9}{20}$

7. **(i)** 1 **(ii)** 1 **(iii)** 18 **(iv)** 0 (trick question!)

   (v) 61 **(vi)** 17 **(vii)** 5 **(viii)** 113

   (ix) 0 **(x)** 10

8. **(i)** $\frac{1}{5}$ **(ii)** $\frac{7}{100}$ **(iii)** $\frac{1}{4}$

   (iv) $\frac{3}{4}$ **(v)** $\frac{19}{20}$ **(vi)** $\frac{3}{50}$

   (vii) $\frac{7}{20}$ **(viii)** $\frac{9}{10}$ **(ix)** $\frac{3}{1000}$ **(x)** $\frac{3}{200}$

9. **(i)** 0.25 **(ii)** 0.75 **(iii)** 0.8 **(iv)** 0.125

   (v) 0.4 **(vi)** 0.375 **(vii)** 0.2

   (viii) 0.5 **(ix)** 0.875 **(x)** 0.6

10. **(i)** 0.66 **(ii)** 0.1

SECTION 6

PERCENTAGES

'Percentage' means **'per 100'** (**Latin :** ***centum*** means '**hundred'**.)

It is just a **fraction** with a **denominator** of **100**.

$$\mathbf{100\% = \frac{100}{100} = 1 \text{ whole.}}$$

The **symbol** for **percentage** is **%,** which is a **'lazy'** way of writing $\frac{\quad}{\mathbf{100}}$.

Thus, **1%** is correctly written $\frac{\mathbf{1}}{\mathbf{100}}$,

10% is $\frac{\mathbf{10}}{\mathbf{100}} = \frac{\mathbf{1}}{\mathbf{10}}$,

- and so on.

Then, **finding a percentage** of a quantity is just **finding a fraction** of the quantity.

To find a **fraction of a quantity**, remember to **divide by the bottom number (i**.**e**. **the denominator**) **and multiply by the top number (i**.**e**. **the numerator**).

Since **percentages** are **hundredths and decimal numbers with 2 decimal places are hundredths**, **percentages** are the **same as decimal numbers with 2 decimal places**.

For example:

25% is the **same as 0.25**,
30% is the **same as 0.30** (or 0.3),
3% is the **same as 0.03**
and so on.

1. Finding a Percentage of a Quantity.

Eg. (i) Find 10% of 30 boys.

Method: $10\% = \mathbf{\frac{10}{100}} = \mathbf{\frac{1}{10}}$

$\mathbf{\frac{1}{10}}$ of 30 boys = **3 boys**.

Eg. (ii) Find 25% of 16 girls.

Method: $25\% = \mathbf{\frac{25}{100}} = \mathbf{\frac{1}{4}}$

$\mathbf{\frac{1}{4}}$ of 16 girls = **4 girls**.

Eg. (iii) Find 35% of £160.

Method: $35\% = \mathbf{\frac{35}{100}} = \mathbf{\frac{7}{20}}$

Working out:

$$2\cancel{0}\ \overline{)\,16\cancel{0}}$$

$$\begin{array}{r} 8 \\ \underline{\times\ 7} \\ 56 \end{array}$$

Therefore, $\mathbf{\frac{7}{20}}$ of £160 = **£56**.

2. Expressing One Quantity as a Percentage of Another Quantity.

Write the **first quantity over second** and **multiply by** $\mathbf{\frac{100}{1}}$.

Eg. (i) Write 2 boys as a percentage of 5 boys.

Method: $\mathbf{\frac{2}{5}} \times \mathbf{\frac{100}{1}} = \mathbf{\frac{200}{5}} = \mathbf{40\%}$.

Eg. (ii) There are 9 white cats and 3 black cats in a pet show. What percentage of the cats are white ?

Method: $\mathbf{\frac{9}{12}} \times \mathbf{\frac{100}{1}} = \mathbf{\frac{900}{12}} = \mathbf{75\%}$.

Eg. (iii) In a tray of bulbs there are 16 daffodils, 8 narcissi and 24 pansies.

(a) What percentage of the bulbs are daffodils?

Method: $\frac{16}{48} \times \frac{100}{1} = \frac{1600}{48} = \frac{100}{3} = 33\frac{1}{3}\%$.

(b) What percentage of the bulbs are narcissi?

Method: $\frac{8}{48} \times \frac{100}{1} = \frac{800}{48} = \frac{100}{6} = \frac{50}{3} = 16\frac{2}{3}\%$.

(c) What percentage of the bulbs are pansies?

Method: $\frac{24}{48} \times \frac{100}{1} = \frac{2400}{48} = \frac{100}{2} = 50\%$.

(Check: $33\frac{1}{3}\% + 16\frac{2}{3}\% + 50\% = 100\% =$ **1 Whole)**.

EXERCISE 6

1. Write the following **percentages** as **fractions** in their lowest terms:

(i) 5% **(ii)** 18% **(iii)** 30% **(iv)** 50%
(v) 20%

2. Write the following **percentages** as **decimal numbers**:

(i) 5% **(ii)** 15% **(iii)** 90% **(iv)** 35%
(v) 6%

3. Find the following **quantities**:

(i) 8% of 100 children **(ii)** 3% of 1000 birds
(iii) 20% of £20.00 **(iv)** 15% of 60 men
(v) 70% of 200 women

4. Write the following as **percentages**:

(i) 2 men out of 5 men
(ii) £3.00 out of £10.00
(iii) 4 eggs out of 20 eggs
(iv) 6 pence out of £1.00 (careful !)
(v) 3 marbles out of 6 marbles.

5. In a P.7 class, **45%** of the pupils are **girls**. If there are **20** children in the class:

(i) How many girls are there?
(ii) How many boys are there?
(iii) What percentage of the pupils are boys?

6. There are **120 members** in a youth club. **30%** of them are **dark- haired**, $\frac{1}{4}$ are **fair-haired** and **the rest** are **red- haired**.

(i) How many have dark hair?
(ii) How many have fair hair?
(iii) How many have red hair?
(iv) What percentage have fair hair?
(v) What percentage have red hair?
(vi) What fraction have red hair?

EXERCISE 6 - ANSWERS

1. **(i)** $\frac{1}{20}$ **(ii)** $\frac{9}{50}$ **(iii)** $\frac{3}{10}$

   (iv) $\frac{1}{2}$ **(v)** $\frac{1}{5}$

2. **(i)** 0.05 **(ii)** 0.15 **(iii)** 0.9(0)

   (iv) 0.35 **(v)** 0.06

3. **(i)** 8 children **(ii)** 30 birds **(iii)** £4.00

   (iv) 9 men **(v)** 140 women

4. **(i)** 40% **(ii)** 30% **(iii)** 20%

   (iv) 6% **(v)** 50%

5. **(i)** 9 **(ii)** 11 **(iii)** 55%

6. **(i)** 36 **(ii)** 30 **(iii)** 54

   (iv) 25% **(v)** 45% **(vi)** $\frac{9}{20}$

SECTION 7

RATIO – UNEQUAL SHARING

Ratio

Ratio means '**proportion'** and gives a **comparison** between **similar quantities**.
The **units** in each quantity **must be the same** before a ratio can be stated.

The symbol **:** means '**is to**' and is used when stating a ratio.

Eg. **25p** is to **50p** as **1 : 2**.

Eg. 1

A piece of string of length **1m** is cut **into 3 smaller lengths** of **20 cm, 30 cm** and **50 cm**.
Write these **3 smaller lengths** as a **ratio**, in its simplest form.

Method:
We have:

	Length 1	**:**	**Length 2**	**:**	**Length 3**
Lengths in cm.	20	:	30	:	50
¸ **across by 10**	**2**	**:**	**3**	**:**	**5**

Therefore, the required **ratio** is **2 : 3 : 5**.

Eg. 2

3 children, Anne, Ben and Charlie are each given pocket money. If **Anne** gets **95p**, **Ben** gets **£1.00** and **Charlie** gets **80p**, write their amounts of pocket money as a **ratio**, in its simplest form.

Method:

We have:

	A	**:**	**B**	**:**	**C**	
Amounts in Pence	95	:	100	:	80	**NOTE**: **Ben's £1.00 must be changed to pence.**
$\div$ **across by 5**	**19**	**:**	**20**	**:**	**16**	

Therefore, the required **ratio** is **19 : 20 : 16**.

Eg. 3 Write the 3 quantities $\mathbf{1\frac{1}{4}}$, $\mathbf{1\frac{1}{2}}$ and **2** as a **ratio**, in its simplest form.

Method:

This time, we need to **change each number into** $\mathbf{\frac{1}{4}}$**s** in order to get **whole numbers**, before proceeding.

We have:

	1st Quantity	**:**	**2nd Quantity**	**:**	**3rd Quantity**
	$1\frac{1}{4}$	:	$1\frac{1}{2}$	:	2
Changed into $\mathbf{\frac{1}{4}}$**s**	**5**	**:**	**6**	**:**	**8**

Therefore, the required **ratio** is **5 : 6 : 8**.

Unequal Sharing

Ratio is used for **proportional parts** in unequal sharing.

Example A piece of string, **1 m long**, is cut into **2 shorter lengths**, with **1 of these lengths 3 times the other**. Find the **2 shorter lengths.**

We have:

	Length 1	:	**Length 2**	
Shares	3	:	1	= **4 shares** = **1 m**
Length in cm	**75**	**:**	**25**	Therefore, **1 share = 25 cm**
				and **3 shares = 75 cm**

So, the **2 shorter lengths** are **75 cm** and **25 cm**.

Eg. 1 Share **£500** among **A**, **B** and **C** so that for every **£1** that **A** has, **B** has **£2** and **C** has **£5**. Find how much money each gets.

Method:

(i) Set up shares using **ratio method** , letting the one with the **least** money have **1 share.**

(ii) **Add** up the **shares** and **divide** into **total** money to find out what **1 share** is equal to.

(iii) Change **shares** into **money**.

Solution:

We have:

	A	:	**B**	:	**C**	
Shares	1	:	2	:	5	= **8 shares** = **£500**
	$\times$ **£62.50**		$\times$ **£62.50**		$\times$ **£62.50**	
Money	£62.50	:	£125	:	£312.50	

Therefore :- $8\overline{)500.00}$ gives : (quotient **62.50**)

1 share = £62.50, i.e. A's share
2 shares = £125.00, i.e. B's share
and 5 shares = £312.50, i.e. C's share.

Eg. 2 Alan is twice as old as Bob who is three times as old as Clare. Clare and Denise are twins. Their average age is 11 years.
How old is each one ?

Solution:

We have:

If the **average age** of a set of **4** people is **11**, then their **total age** must be **44 years**. **(i.e. 11 ´ 4)**

	A	:	B	:	C	:	D	
Shares	6	:	3	:	1	:	1	= **11 shares** = **44 years**
	$\times 4$	:	$\times 4$	:	$\times 4$	:	$\times 4$	
Years	24	:	12	:	4	:	4	So **1 share** = **4 years**

Therefore,

Alan is **24 yrs old**,
Bob is **12 yrs old**,
Clare is **4 yrs old**,
and **Denise** is **4 yrs old**.

Eg. 3 Tom, Dick and Harry share **70 sweets** so that **Dick gets** $\mathbf{\frac{1}{3}}$ of the number that **Harry** gets, and **Harry gets** $\mathbf{\frac{1}{2}}$ as many as **Tom**.

How many sweets does each boy get?

Solution:

We have:

	T	:	D	:	H		
Shares	6	:	1	:	3	=	**10 Shares = 70 sweets**
	$\times 7$		$\times 7$		$\times 7$		
Sweets	42	:	7	:	21		Then, **1 share = 7 sweets**

Therefore, **Tom** gets **42**, **Dick** gets **7** and **Harry** gets **21** sweets.

Eg. 4

Some pictures are shared among John and Michelle in the **ratio** of their **ages**. **John** is **5** years old and **Michelle** is **3** years **old**.

If **John** gets **15** pictures:

(i) How many pictures does Michelle get?
(**ii**) How many pictures are there altogether?

Solution:

We have:

	J	:	M		
Shares	5	:	3		**5 shares** = **15 pictures**
Pictures	15	:	9	Then,	**1 share** = **3 pictures**
				and	**3 shares** = **9 pictures**

Therefore:

(**i**) **Michelle** gets **9 pictures**.

(**ii**) There are **24 pictures altogether**.

Other Types of Unequal Sharing

Eg. 1

Share **51 marbles** among Jack and Sue so that Sue gets **19 more** than Jack.

How many marbles does each child get?

Solution:

$$\begin{array}{r l} 51 & \\ -\ \underline{19} & \\ 2\ \underline{)\ 32} & \\ \mathbf{16} & = \textbf{Jack's Share} \\ +\ \underline{19} & \\ \mathbf{35} & = \textbf{Sue's Share}. \end{array}$$

Eg. 2

The **sum** of **2 numbers** is **39**, and the **difference** between them is **21**. Find the 2 numbers.

$$\begin{array}{rl} & 39 \\ + & \underline{21} \\ 2\,) & \underline{60} \\ & \mathbf{30} = \textbf{larger number} \\ \text{then} & \mathbf{9} = \textbf{smaller number} \end{array} \qquad \textbf{or} \qquad \begin{array}{rl} & 39 \\ - & \underline{21} \\ 2\,) & \underline{18} \\ & \mathbf{9} = \textbf{smaller number} \\ & \mathbf{30} = \textbf{larger number}. \end{array}$$

Eg. 3

Alan, Betty and Chris share **£18** so that **Alan** gets **twice** as much as **Chris**, and **Betty** gets **£2 more** than **Chris**. How much does each child receive?

Method:

This time, it is convenient to let the smallest amount be **£*x*** and write the **other 2 amounts using *x***.

We have:

	A	:	B	:	C		
Add	$2x$		$x + 2$		x	=	$4x + 2$
This gives	**£8**	:	**£6**	:	**£4**		Then $4x + 2 = 18$ i.e. $x = £4$.

EXERCISE 7

1. Write each of the following as a ratio, in its simplest form:

(i) 25p to 75p
(ii) 60 marbles to 85 marbles
(iii) 18 sweets to 45 sweets
(iv) 45p to £1.15
(v) 250g to 1 kg (Remember, 1000g = 1 kg)
(vi) 30 cm to 1m (Remember, 100 cm = 1m)

(**vii**) 175 mg to 1g (Remember, 1000mg = 1g)
(**viii**) 70mm to 2m (Remember, 1000mm = 1m)
(**ix**) 150 ml to 1*l* (Remember, 1000 ml = 1*l*)
(**x**) £1.20 to £2.10 (Change both into pence first.)

2. Write each of the sets of 3 quantities below as a ratio in its simplest form:

(**i**) 50 marbles, 100 marbles and 150 marbles.
(**ii**) 80p, 70p and £1.20.
(**iii**) 15 cm, 27 cm, and 1.23 m.
(**iv**) 1.5 kg, 700g and 30g.
(**v**) 2*l*, 1.5*l* and 150 ml.
(**vi**) $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$.
(**vii**) $1\frac{1}{2}$, 2 and $2\frac{1}{4}$.

3. A sum of money is shared among A, B and C in the ratio 1 : 2 : 3. If A gets 25p,

(**i**) how much does B get?
(**ii**) how much does C get?
(**iii**) how much money is shared altogether?

4. Share £36 among A, B and C in the ratio 2 : 3 : 4.

(**i**) How much does A get?
(**ii**) How much does B get?
(**iii**) How much does C get?

(Check that your answers to parts (i), (ii) and (iii) total £36.)

5. Share £24.00 among Ann, Ben and Con so that Ann gets twice as much as Ben, and Con gets 3 times as much as Ben.

(**i**) How much does Ann get?
(**ii**) How much does Ben get?
(**iii**) How much does Con get?

(Firstly, set up shares as a ratio, letting the child with the least amount have 1 share. Then proceed as before.)

6. Share 39 sweets among Mary and Peter, letting Mary have 3 more than Peter.

How many does:
- (**i**) Mary get?
- (**ii**) Peter get?

7. The sum of 2 numbers is 51 and the difference between them is 19. Find the 2 numbers.

8. 38 marbles are shared among Dan, Ed and Flo so that Dan gets twice as many as Ed and Flo gets 2 more than Ed.

How many marbles does:
- (**i**) Dan get?
- (**ii**) Ed get?
- (**iii**) Flo get?

(Use *x* for smallest and write others using *x*. Then proceed.)

9. A 1m length of tape is cut into 2 parts so that the first part is 4 times the length of the second part. Find the length (in cm) of:
- (**i**) the first part.
- (**ii**) the second part.

(Set up 2 parts as a ratio, letting the smaller part have 1 share.)

10. Gina, Helen and Ian share 18 sweets so that Gina gets $\frac{1}{2}$ of the number that Helen gets, and Helen gets $\frac{1}{3}$ of the number of Ian. How many does:
- (**i**) Gina get?
- (**ii**) Helen get?
- (**iii**) Ian get?

(Again, set up shares as a ratio and then proceed.)

EXERCISE 7 - ANSWERS

1. (i) 1:3 (ii) 12:17 (iii) 2:5

   (iv) 9:23 (v) 1:4 (vi) 3:10

   (vii) 7:40 (viii) 7:200 (ix) 3:200

2. (i) 1:2:3 (ii) 8:7:12 (iii) 5:9:41

   (iv) 150:70:3 (v) 40:30:3

   (vi) 2:1:3 (vii) 6:8:9

3. (i) 50p (ii) 75p (iii) £1.50

4. (i) £8.00 (ii) £12.00 (iii) £16.00

5. (i) £8.00 (ii) £4.00 (iii) £12.00

6. (i) 21 (ii) 18

7. 35 and 16

8. (i) 18 (ii) 9 (iii) 11

9. (i) 80cm (ii) 20cm

10. (i) 2 (ii) 4 (iii) 12

SECTION 8

FOREIGN EXCHANGE

Each country has its own money which is called its **currency**. The currency of the United Kingdom is in pounds **sterling**.

It is possible to change one currency into another if we know the **rate of exchange**,

Eg. £1.00 sterling = 1.44 Euro,
or £1.00 sterling = 2.04 United States Dollars.

Worked Example

1. If £1 is worth 2.10 U.S. dollars work out the following:

(a) How many dollars would £5.50 be worth?
(b) Give the value of 96 dollars in £ s.
(c) What would 256 dollars be worth in £ s?
(d) Change £2.60 into dollars.

Hint:
Do out a **conversion table**, starting with the rate of exchange, to help you to answer the questions, like this:

£	**:**	**Dollars**
1.00	:	2.10
100.00	:	210.00
0.50	:	1.05
50.00	:	105.00
0.10	:	0.21
10.00	:	21.00

ANSWERS

(a) £5.50 = $(2.10 \times 5) + 1.05$ = **11.55 dollars**
(b) 315 dollars = £100.00 + £50.00 = **£150.00**
(c) 336 dollars = £100.00 + £50.00 + £10.00
= **£160.00**
(d) £2.60 = 2.10 + 2.10 + 1.05 + 0.21
= **5.46 dollars**.

Generally, there are **2 rules** which can be employed when **changing between our currency and any foreign currency**:

Rule 1

When **changing sterling into a foreign currency, multiply by the rate of exchange**.

E.g. If £1.00 = 2.50 Australian dollars,
converting £5.00 to **Australian dollars**, we have:
5 ´ 2.50 = 12.50 Australian dollars.

Rule 2

When **changing a foreign currency into sterling, divide by the rate of exchange**.

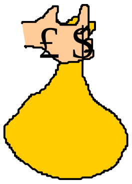
E.g. If **£1.00 = I 1.50**,
converting 60 euro to £ sterling, we have:
60 ÷ 1.50
= **£40.00**.

EXERCISE 8

£ $ £ $ £ $ £ $ £ $

Country	Currency	Exchange Rate per £ sterling	
Australia	Dollar	Aus.	2.40
Canada	Dollar	Can.	2.05
Cyprus	Pound	Cyp.	0.85
Czech Republic	Koruna		40
Denmark	Krone		10.70
Egypt	Pound	Egypt	11.30
European Community	Euro		1.45
Hong Kong	Dollar	Hong Kong	15.70
Hungary	Forint		360
India	Rupee		80
Israel	Shekel		8.20
Japan	Yen		230
Malta	Pound	Malta	0.60
Mexico	Peso		22
New Zealand	Dollar	N.Z.	2.70
Norway	Krona	Norway	11.20
Singapore	Dollar	Singapore	3
South Africa	Rand		14.30
Sweden	Krona	Sweden	13.30
Switzerland	Franc		2.40
Thailand	Baht		65
Turkey	New Lira		2.50
United Arab Emirates	Dirham		7.40
United States of America	Dollar	U.S.A.	2.00

1. Using the **Exchange Rates** given in the **table above**, do the following **conversions**:



(i) £400 to Swiss francs.
(ii) 480 Swiss francs to £ sterling.
(iii) £700 to United States dollars.
(iv) 480 United States dollars to £ sterling.
(v) £500 sterling to Cypriot pounds.
(vi) 255 Cypriot pounds to £ sterling.
(vii) £9 to Australian dollars.
(viii) 24,000 Australian dollars to £ sterling.
(ix) £5,000 sterling to Swedish krona.
(x) 3,990 Swedish krona to £ sterling.
(xi) £250 sterling to Thai baht.
(xii) 3,250 Thai baht to £ sterling.
(xiii) £500 sterling to Turkish new lira.
(xiv) 1,000 Turkish new lira to £ sterling.
(xv) £500 to Singaporean dollars.
(xvi) 293,550 Singaporean dollars to £ sterling.
(xvii) £30,600 to Maltese pounds.
(xviii) £500 to Israeli shekel.
(xix) 26,240 Israeli shekel to £ sterling.
(xx) 216,000 Hungarian forint to £ sterling.
(xxi) £216 to Hungarian forint.
(xxii) 57,500 Japanese yen to £ sterling.
(xxiii) £500 to Japanese yen.
(xxiv) 29,000 euro to £ sterling.
(xxv) £5,800 to euro.
(xxvi) 16,950 Egyptian pounds to £ sterling.
(xxvii) £2,260 to Egyptian pounds.
(xxviii) 1,070 Danish krone to £ sterling.
(xxix) £535 to Danish krone.
(xxx) 30,000 Czech Republican koruna to £ sterling.
(xxxi) £7,500 to Czech Republican koruna.
(xxxii) 30,750 Canadian dollars to £ sterling.
(xxxiii) £123,000 to Canadian dollars.
(xxxiv) 3,925 Hong Kong dollars to £ sterling.
(xxxv) £15,700 to Hong Kong dollars.
(xxxvi) 3,200 Indian rupees to £ sterling.
(xxxvii) £3,200 to Indian rupees.

(xxxviii) 3,300 Mexican peso to £ sterling.
(xxxix) £3,300 to Mexican peso.
(xl) 10,800 New Zealand dollars to £ sterling.
(xli) £2,000 to New Zealand dollars.
(xlii) 4,480 Norwegian krona to £ sterling.
(xliii) £2,000 to Norwegian krona.
(xliv) 4,290 South African rand to £ sterling.
(xlv) £450 to South African rand.
(xlvi) 14,800 United Arab Emirates dirham to £ sterling.
(xlvii) £70 to United Arab Emirates dirham.

2. A Lismoghan family are going on holiday to Switzerland for 21 days and they calculate that they will need £2,500 worth of Swiss francs to cover their expenses in Switzerland.

(i) How many **Swiss francs** do they receive from the bank in exchange for their **£2,500**?

Answer . . . .

(ii) Whilst in Switzerland, they spend an average of **275 francs** per day for the **21 days**.
Find, **in francs**, how much they spend altogether.

Answer . . . .

(iii) On their return to Lismoghan, they change the **remainder** of their francs **back into £ sterling**.
(a) How many **francs** do they have on their return?

Answer . . . .

(b) How much, **in sterling**, do they receive from the bank, in exchange for the francs returned?

Answer . . . .
(Use the exchange rate given in the table earlier.)

3. A Belfast family are going on holiday to Spain for 25 days and they calculate that they will need £2,500 worth of euro to cover their expenses in Spain.

(i) How many **euro** do they receive from the bank in exchange for their **£2,500**?

(ii) Whilst in Spain, they spend an average of **130 euro** per day for the **25 days**.
Find, **in euro**, how much they spend altogether.

(iii) On their return to Belfast, they change the **remainder** of their euro **back into £ sterling**.

(a) How many **euro** do they have on their return?

(b) How much, **in sterling**, do they receive from the bank, in exchange for the euro returned?

EXERCISE 8 - ANSWERS

4. **(i)** 960 francs.
(ii) £200.
(iii) 1,400 dollars.
(iv) £240.
(v) 425 pounds.
(vi) £300.
(vii) 21.60 dollars.
(viii) £10,000.
(ix) 66,500 krona.
(x) £300.
(xi) 16,250 baht.
(xii) £50.
(xiii) 1,250 new lira.
(xiv) £400.
(xv) 1,500 dollars.
(xvi) £97,850.
(xvii) 18,360 ponnds.
(xviii) 4,100 shekel.
(xix) £3,200.
(xx) £600.
(xxi) 67,760 forint.
(xxii) £250.
(xxiii) 115,000 yen.
(xxiv) £20,000.
(xxv) 8,410 euro.
(xxvi) £150.
(xxvii) 25,538 pounds.
(xxviii) £100.
(xxix) 5,724.50 krone.
(xxx) £750.
(xxxi) 300,000 koruna.
(xxxii) £15,000.
(xxxiii) 252,150 dollars.
(xxxiv) £250.
(xxxv) 246,490 dollars.
(xxxvi) £40.

(xxxvii) 256,000 rupees.
(xxxviii) £150.
(xxxix) 72,600 peso.
(xl) £4,000.
(xli) 5,400 dollars.
(xlii) £400.
(xliii) 22,400 krona.
(xliv) £300.
(xlv) 6,435 rand.
(xlvi) £2,000.
(xlvii) 518 dirham.

5. **(i)** 6,000 francs.

   (ii) 5,775 francs.

   (iii) **(a)** 225 francs. **(b)** £93.75.

6. **(i)** 3,625 euro.

   (iii) 3,250 euro.

   (iii) **(a)** 375 euro. **(b)** £258.62 (correct to the nearest penny).

SECTION 9

AVERAGE SPEED , DISTANCE & TIME
- TRAVEL GRAPHS

AVERAGE SPEED , DISTANCE & TIME

Average Speed is **km per hour, metres per second,** etc**.**

1. **SPEED = DISTANCE ÷ TIME**

E.g. A car travels 100 km in 2 hours. Find its speed.
Speed of Car $= 100 \div 2$
$=$ **50 km/hr**.

2. **DISTANCE = SPEED × TIME**

E.g. A car travels for $2\frac{1}{2}$ hours at 50 km/hr. How far does it travel ?

Distance travelled $= 50 \times 2\frac{1}{2}$
= 125 km.

3. **TIME = DISTANCE ÷ SPEED**

E.g. A car travels a distance of 60 km at a speed of 80 km/hr.
How long does the journey take ?

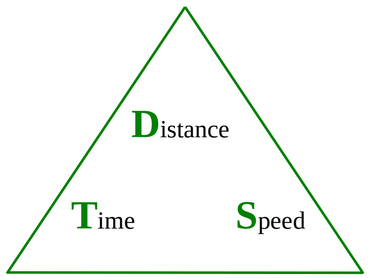
Time taken (in **hours**) is $60 \div 80$

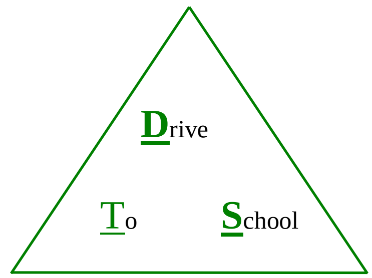
$$= \frac{60}{80}$$

$$= \frac{3}{4}$$

$=$ **45 mins**.

The triangle below serves as a good **mnemonic**
(i.e. aid to memorisation) for **Speed**, **Distance** and **Time.**





To **use** this triangle, **cover up the quantity you are required to find** and, thus, read:

	$\mathbf{D = TS}$,	i.e.	**Distance**	=	**Time ´ Speed**
	$\mathbf{T = \dfrac{D}{S}}$,	i.e.	**Time**	=	**Distance ¸ Speed**
and	$\mathbf{S = \dfrac{D}{T}}$.	i.e.	**Speed**	=	**Distance ¸ Time**

Worked Examples

1. A car travels 150 km in $2\frac{1}{2}$ hours. Find its speed.

2. A car travels for 3 hrs. at 70 km/hr. How far does it travel?

3. A car travels a distance of 100 km at a speed of 80 km/hr. How long does the journey take?

4. John and Jackie live 24 km apart. John runs at 6 km/hr. and Jackie cycles at 8 km/hr. They decide to meet at a point mid-way between their homes.

(a) If John leaves home at 10.51 am, at what time does he reach the half-way point?

(b) At what time does Jackie have to leave home in order to arrive at the half-way point at the same time as John?

One evening the children arrange to meet at the half-way point at 6.40 p.m.

(c) At what time does John have to leave home if he increases his speed to 8 km/hr?

(d) At what time must Jackie leave home if she increases her speed to 9 km/hr?

The children spend 35 mins. together at the half-way point and then set off for their respective homes.

(e) If John must be home at 8.35 p.m., at what speed must he run from the half-way point?

(f) If Jackie must be home at 8.27 p.m., at what speed must she cycle from the half-way point?

ANSWERS

1. Speed of Car

$$= 150 \div 2\frac{1}{2}$$

$$= \frac{150}{1} \div \frac{5}{2} \qquad = \frac{150}{1} \times \frac{2}{5} \qquad =$$

$$= \mathbf{60\ km/hr}.$$

2. Distance Travelled

$$= 70 \times 3 \qquad = \mathbf{210\ km}.$$

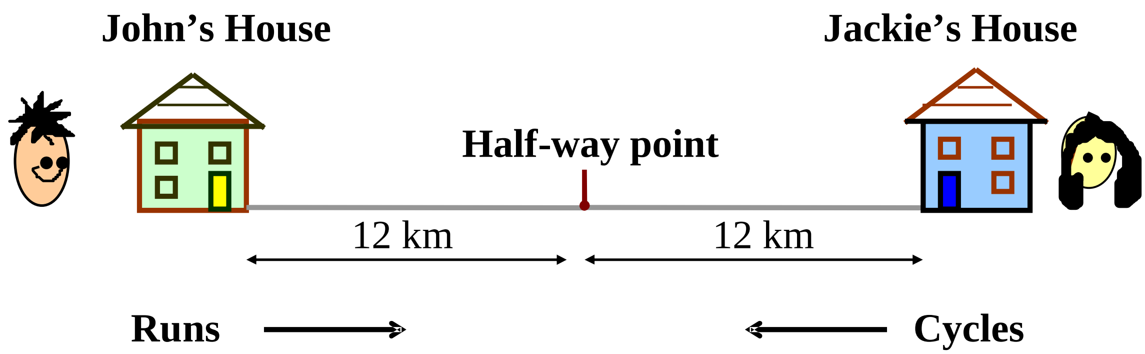
3. Time taken

$$= 100 \div 80$$

$$= \frac{100}{80} \qquad = 1\frac{20}{80} \qquad = 1\frac{1}{4} \text{ hrs.}$$

$$= \mathbf{1\ hr.\ 15\ min}.$$

4. It is helpful to **draw a diagram** when answering a question of this type. The **diagram** looks like this:



(a) The **distance** to the half-way point is
24 km ÷ 2 = **12 km**.

We know that John's **speed** is 6 km/hr.

We are required to find the **time** taken.

Time = Distance ÷ Speed
= 12 ÷ 6
= **2 hr**.

Then

	Hr	Min
	10	51
+	2	00
	12	**51**

This means that John reaches half-way point at **12.51 p.m**.

(b) Jackie also has to travel a **distance** of **12 km** and her **speed** is 8 km/hr.

Again, we are required to find the **time** taken.

Time = Distance ÷ Speed
= 12 ÷ 8
= $\mathbf{1\frac{1}{2}}$ **hr**.

Then

	Hr	Min
	12	51
-	1	30
	11	**21**

This means that Jackie must leave home at **11.21 a.m**. in order to reach the half-way point at 12.51 p.m.

(c) **Distance** = **12 km**
John's **speed** = **8 km/hr**

Time taken = Distance ÷ Speed
= 12 ÷ 8
= $\mathbf{1\frac{1}{2}}$ **hr = 1 hr 30 min**.

Then

	Hr	Min
	6	40
-	1	30
	5	**10**

This means that John must leave home at **5.10 p.m**. in order to reach the half-way point at 6.40 p.m.

(d) **Distance** = **12 km**
Jackie's **speed** = **9 km/hr**.

Time taken = Distance ÷ Speed
= 12 ÷ 9
= $\mathbf{1\frac{1}{3}}$ **hr = 1 hr 20 min**.

Then

	Hr	Min
	6	40
-	1	20
	5	**20**

This means that Jackie must leave home at **5.20 p.m.** in order to reach the half-way point at 6.40 p.m.

Since

	Hr	Min
	6	40
+		35
	7	**15**

This means that each child leaves half-way point at **7.15 p.m**.

(e) If John must be home at 8.35 p.m., his time for the journey home is:

	Hr	Min
	8	35
-	7	15
	1	**20**

i.e. **1 hr 20 min**

Then, we have:

Distance = **12 km**

Time = **1 hr 20 min** = $1\frac{20}{60}$ = $\mathbf{1\frac{1}{3}}$ **hr**.

Now, we are required to find the **speed**.

Speed = Distance $\div$ Time

$$= 12 \div 1\frac{1}{3}$$

$$= \frac{12}{1} \div \frac{4}{3} = \frac{12}{1} \times \frac{3}{4}$$

$$= \frac{36}{4}$$

= **9 km/hr**.

So, if John runs at a speed of **9 km/hr**, he will reach his home at 8.35 p.m.

(f) If Jackie must be home at 8.27 p.m., her time for the journey home is

	Hr	Min
	8	27
-	7	15
	1	**12**

i.e. **1 hr 12 min**.

Then, we have:

Distance = **12 km**

Time = 1 hr 12 min $= 1\frac{12}{60} = \mathbf{1\frac{1}{5}}$ **hrs**

We are required to find the **speed**.

Speed = Distance ÷ Time

$$= 12 \div 1\frac{1}{5}$$

$$= \frac{12}{1} \div \frac{6}{5}$$

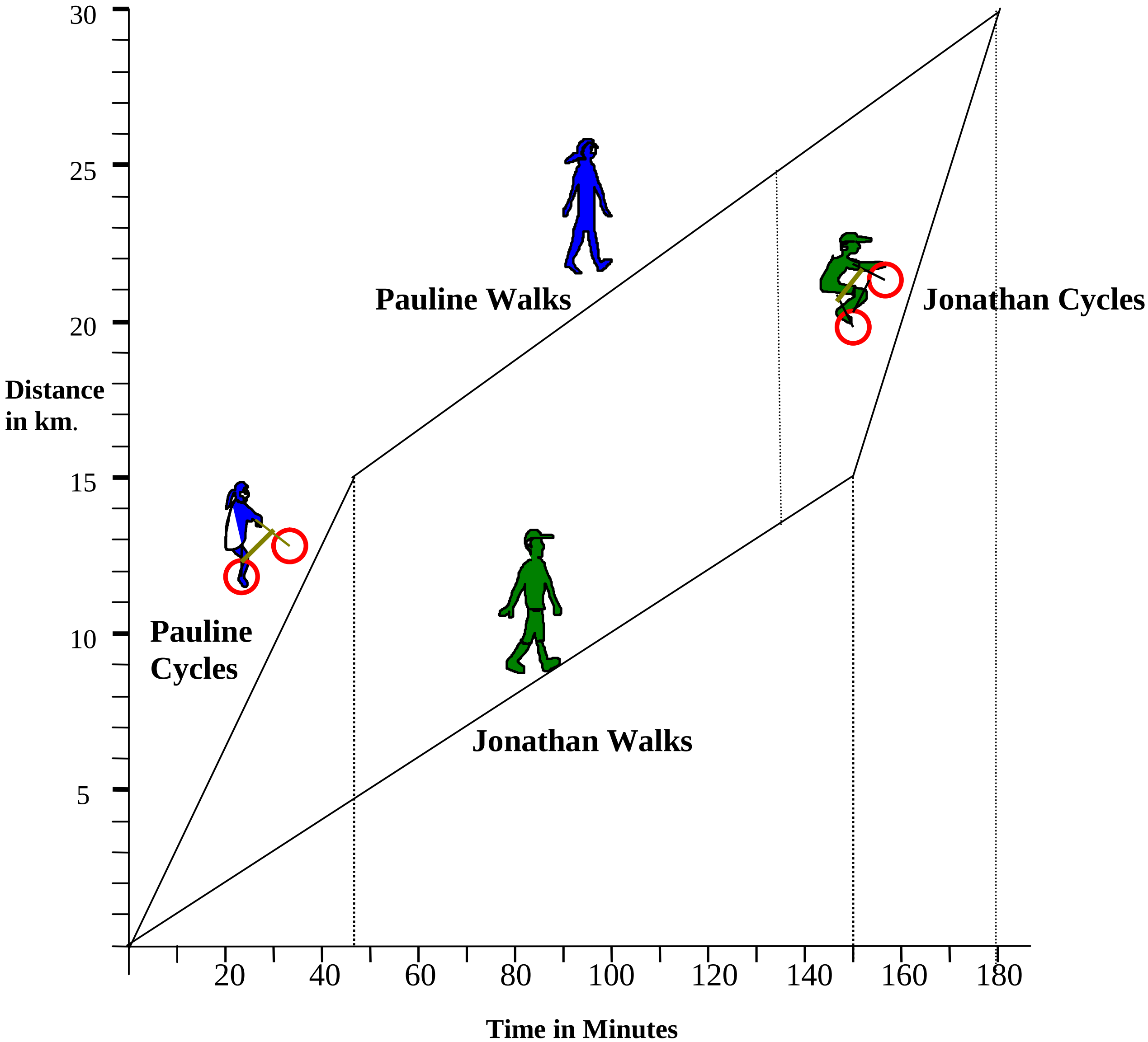
$$= \frac{12}{1} \times \frac{5}{6}$$

$$= \frac{60}{6}$$

= **10 km/hr**.

So, if Jackie cycles at a speed of **10 km/hr**, she will reach her home at 8.27 p.m.

1. The journeys of Pauline and Jonathan



Pauline and Jonathan set out together to go from Dundrum to Newry, 30 km away. Firstly, Pauline cycled and Jonathan walked. When Pauline had cycled half the distance, she left the bicycle by the roadside at Stang for Jonathan to use when he reached it. Pauline completed the journey by walking. The graph above illustrates their journeys.

(a) They left Dundrum at 10.00 a.m.
At what time did they arrive in Newry?

(b) For how long did Pauline cycle?

(c) What was Pauline's average cycling speed in km/hr?

(d) For how long was the bicycle lying beside the road at Stang?

(e) When Pauline left the bicycle, how far had Jonathan walked?

(f) How far were Pauline and Jonathan apart after $2\frac{1}{4}$ hours?

(g) What was Jonathan's average cycling speed in km/hr?

(h) What was Pauline's average speed in km/hr for the whole journey?

ANSWERS

(a) The whole time is **180 min** **=** 180 ÷ 60 = **3 hr**.

Then		Hr	Min
		10	00
	+	3	00
		13	**00**

= **1.00 p.m**.

(b) **45 minutes**.

(c) Pauline cycled a distance of 15 km in 45 min.

Then: **Distance** = **15 km**.

Time = $\frac{45}{60}$ = $\mathbf{\frac{3}{4}}$ **hr**.

Speed = Distance ÷ Time

$$= 15 \div \frac{3}{4}$$

$$= \frac{15}{1} \times \frac{4}{3}$$

$$= \frac{60}{3}$$

= **20 km/hr**.

(d) Pauline left the bicycle at Stang **45** minutes after leaving Dundrum and Jonathan reached the bicycle **150** minutes after leaving Dundrum.

Therefore, the time that the bicycle was left by the roadside is

$$\begin{array}{r} 150 \\ -\underline{\ \ 45} \\ 105 \text{ minutes} \end{array} \quad = \quad \textbf{1 hour 45 minutes}.$$

(e) Pauline left the bicycle after 45 minutes.

After 45 min, Jonathan had walked approximately **4.4 km**.

(f) $2\frac{1}{4}$ hrs $\quad = \quad 2\frac{1}{4} \times 60 \quad = \quad$ **135 minutes**.

The **distance** between them after 135 minutes is

25 km - 13.5 km **=** **11.5 km**.

(g) Jonathan cycled a distance of 15 km in 30 min

= **30 km/hr**.

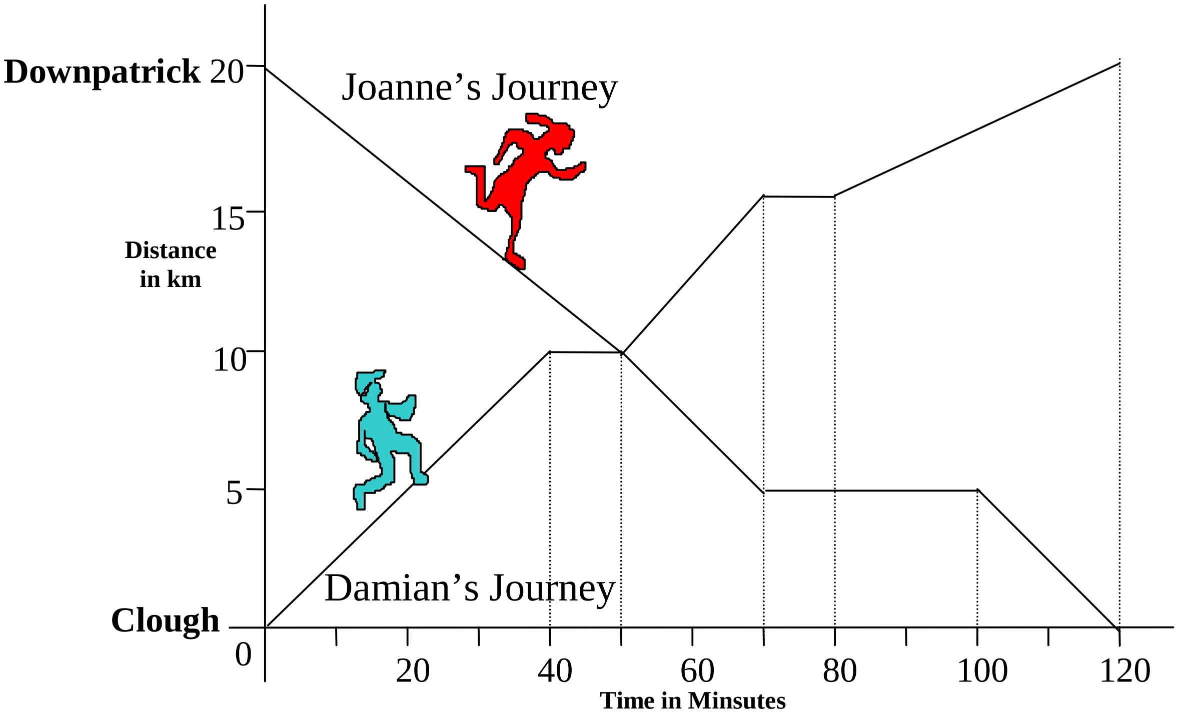
(h) Pauline travelled a total distance of **30 km** in 3 hr.

Speed = Distance $\div$ Time

= 30 $\div$ 3

= **10 km/hr**.

2. The Journeys of Damian and Joanne.



Damian and Joanne start jogging at 9.30 a.m. Damian’s journey is from Clough to Downpatrick and Joanne’s journey is from Downpatrick to Clough. The distance between Downpatrick and Clough is 20 km. The graph above illustrates their journeys.

(a) For how many minutes was Damian actually running?

(b) For how many minutes was Damian stopped?

(c) For how many minutes was Joanne actually running?

(d) For how many minutes was Joanne stopped?

(e) How far was Joanne from Downpatrick when she passed Damian?

(f) At what time did the children meet?

(g) What was Damian's jogging speed before he took a rest?

(h) What was Joanne's jogging speed between Downpatrick and the point where she met Damian?

(i) What was Joanne's jogging speed just before she rested?

(j) What was Damian's jogging speed between his resting periods?

(k) What was Damian's jogging speed just before he reached Downpatrick?

(l) At what time did the children finish jogging?

ANSWERS

(a) 40 minutes + 20 minutes + 40 minutes = **100 minutes**.

(b) 10 minutes + 10 minutes = **20 minutes**.

(c) 50 minutes + 20 minutes + 20 minutes = **90 minutes**.

(d) **30 minutes**.

(e) **10 km**.

(f) After 50 minutes, then

Hr	Min
9	30
+	50
10	**20**

(g) **Distance** is **10 km** and **Time** is $\frac{40}{60} = \mathbf{\frac{2}{3}}$ **hr**.

$$\text{Speed} = \text{Distance} \div \text{Time}$$
$$= \frac{10}{1} \div \frac{2}{3}$$
$$= \frac{10}{1} \times \frac{3}{2}$$
$$= \mathbf{15\ km/hr}.$$

(h) **Distance** is **10 km** and **Time** is $\frac{50}{60} = \mathbf{\frac{5}{6}}$ **hr**.

Speed = Distance ÷ Time

$$= \frac{10}{1} \div \frac{5}{6}$$

$$= \frac{10}{1} \times \frac{6}{5} = \mathbf{\frac{60}{5}}$$

$$= \textbf{12 km/hr}.$$

(i) **Distance** is **5 km** and **Time** is $\frac{20}{60} = \mathbf{\frac{1}{3}}$ **hr**.

Speed = Distance ÷ Time

$$= \frac{5}{1} \div \frac{1}{3}$$

$$= \frac{5}{1} \times \frac{3}{1} = \frac{15}{1}$$

$$= \textbf{15 km/hr}.$$

(j) **Distance** is **5 km** and **Time** is $\frac{20}{60} = \mathbf{\frac{1}{3}}$ **hr**.

Speed = Distance ÷ Time

$$= \frac{5}{1} \div \frac{1}{3}$$

$$= \frac{5}{1} \times \frac{3}{1} = \frac{15}{1}$$

$$= \textbf{15 km/hr}.$$

(k) **Distance** is **5 km** and **Time** is 40 minutes $= \frac{40}{60} = \mathbf{\frac{2}{3}}$ **hr**.

Speed $= \frac{5}{1} \div \frac{2}{3}$

$$= \frac{5}{1} \times \frac{3}{2} = \frac{15}{2}$$

$$= \mathbf{7\frac{1}{2}} \textbf{ km/hr}.$$

(l) Their journeys lasted **120 minutes** = **2 hr**:

$$\begin{array}{rl} & 9.30 \text{ a.m.} \\ + & 2.00 \\ \hline = & \mathbf{11.30 \text{ a.m.}} \\ \hline \end{array}$$

EXERCISE 9

1. Copy and complete the following table:

	AVERAGE SPEED	DISTANCE	TIME TAKEN
(i)	**50 km/hr**	**25 km**	....
(ii)	**38 km/hr**	....	**2 hr**
(iii)	....	**75 km**	**1 hr 30 min**
(iv)	**40 km/hr**	**10 km**	....
(v)	**30 m/sec**	....	**10 sec**
(vi)	....	**12 km**	**20 min**
(vii)	**30 km/hr**	**45 km**	....
(viii)	**16 m/sec**	....	**5 sec**
(ix)	....	**80 km**	**1 hr 20 min**
(x)	**30 km/hr**	**18 km**	....

2. A girl lives 6 km away from her school, which commences at 9.15 a.m. Sometimes she cycles and sometimes she walks to school.

Her cycling speed is 10 km/hr and her walking speed is 5 km/hr.

(i) At what time must she leave home on the bicycle if she is to reach the school on time?

(ii) At what time must she leave home, if she has to walk, in order to arrive on time?

(iii) One morning, as she is leaving home at her normal time by bicycle, she suddenly remembers that she has to stop at the shop to buy a ruler. If she is stopped for 6 minutes altogether, at what speed must she cycle in order to reach the school on time?

(iv) One morning, as she is leaving home at her normal time for walking, she decides to run to school at an average speed of 8 km/hr, so as to arrive at school early for a game of tennis. Calculate how many minutes she will have for tennis before school commences?

(v) At what speed would she have to cycle if she wished to complete the journey in 24 minutes?

(vi) At what speed would she have to walk if she wished to complete the journey in 48 minutes?

3. Two brothers, Conor and Cormac, stay overnight with their separate friends : Conor's in Castlewellan and Cormac's in Seaforde. There is a distance of 10 km between Castlewellan and Seaforde.
The two brothers arrange to leave their respective friends' homes at 9.00 a.m. the following day, with Conor cycling towards Seaforde and Cormac jogging towards Castlewellan.

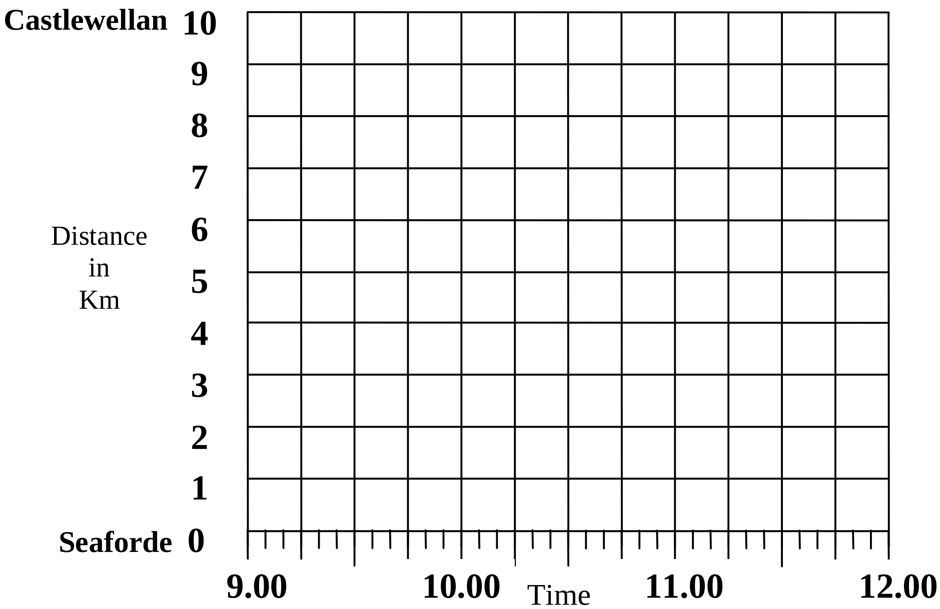
Conor's journey

Conor leaves Castlewellan at 9.00 a.m. and cycles towards Seaforde at an average speed of 12 km/hr.
After 20 mins., he rests for 30 mins and then continues cycling at the same speed for a further 3 km.
He stops for 45 mins. to mend a puncture and then continues cycling towards Seaforde at half his earlier speed.

Cormac's journey

Cormac leaves Seaforde at 9.00 a.m. and jogs towards Castlewellan at an average speed of 8 km/hr.
After 30 mins, he rests for 15 mins and then continues jogging at the same speed for a further 2 km.
Then he meets his old friend, Finnian and they talk for 1 hr at the roadside.
He leaves his friend and decides to walk the remainder of the distance to Castlewellan.
He reaches Castlewellan at 12.00 noon.

(a) Using the information given, draw travel graphs, showing the journeys of Conor and Cormac.



(b) **Using your graph**, answer the following questions:

(i) How far from Seaforde is Cormac when he stops for his rest?

(ii) How far from Castlewellan is Cormac when he meets Finnian?

(iii) What is Cormac's walking speed for the last part of his journey to Castlewellan?

(iv) What is Cormac's average speed over the whole journey?
(Remember: **total** distance $\div$ **total** time)

(v) How far from Castlewellan is Conor when he stops for his rest?

(vi) At what time do Conor and Cormac meet?

(vii) How far from Seaforde does Conor meet Cormac?

(viii) Do you think Conor sees Cormac's old friend, Finnian, on his journey? Give a reason for your answer.

(ix) At what time does Conor stop to mend the puncture?

(x) What is Conor's speed for the last part of his journey?

(xi) At what time does Conor reach Seaforde?

(xii) What is Conor's average speed over the whole journey?
(Again, **total** distance $\div$ **total** time)

Exercise 9 - Answers

1. **(i)** $\frac{1}{2}$ hour (or 30 min) **(ii)** 76 km

(iii) 50 km/hr **(iv)** $\frac{1}{4}$ hour (or 15 min)

(v) 300 m. **(vi)** 36 km/hr.

(vii) $1\frac{1}{2}$ hr **(viii)** 80 m **(ix)** 60 km/hr

(x) $\frac{3}{5}$ hour (or 36 minutes)

2. **(i)** 8.39 a.m. **(ii)** 8.03 a.m.

(iii) 12 km **(iv)** 27 minutes

(v) 15 km/hr **(vi)** $7\frac{1}{2}$ km/hr.

3. **(i)** 4 km **(ii)** 4 km

(iii) 4 km/hr **(Check this on graph)** **(iv)** $3\frac{1}{3}$ km/hr **(Again, check this on the graph)**

(v) 4 km. **(vi)** 9.55 a.m.

(vii) 5 km. **(viii)** Probably not.

(ix) 10.05 a.m. **(x)** 6 km/hr

(xi) 11.20 a.m. **(xii)** $4\frac{2}{7}$ km/hr.

SECTION 10

SHAPES – MEASUREMENT & SYMMETRY

1. TRIANGLE

- **3-sided figure**
- **angles add up to 180°**
- **area** $= \frac{1}{2}$ **base** $\times$ **perpendicular height**.

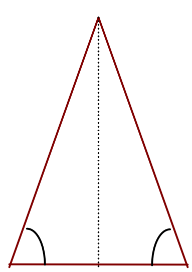
TYPES OF TRIANGLE

(i) Scalene Triangle

- 3 sides of **different** lengths
- 3 **different-sized** angles
- **no line** symmetry
- **rotational** symmetry of **Order 1**

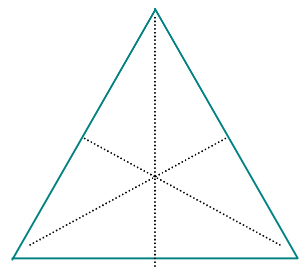
(ii) Isosceles Triangle

- 2 **equal** sides
- 2 **equal-sized** angles
- **1 line** of symmetry (Broken line on diagram)
- **rotational symmetry** of **Order 1**



(iii) Equilateral Triangle

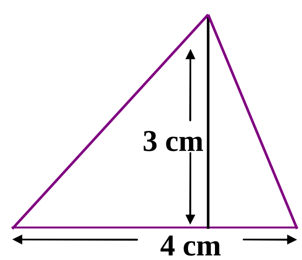
- 3 **equal** sides
- 3 **equal-sized** angles, each 60°.
- **3 lines** of symmetry (Broken lines on diagram)
- **rotational** symmetry of **Order 3**



AREA OF TRIANGLE

Area = $\frac{1}{2}$ **base** ´ **perpendicular height**.

Example



Area of Triangle shown

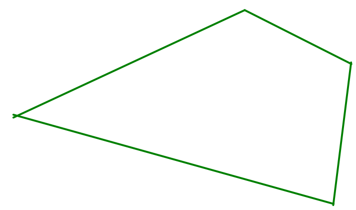
is $\frac{1}{2}$ of 4 cm $\times$ 3 cm

= $\frac{1}{2}$ of 12 cm^2

= **6 cm^2**

2. QUADRILATERAL

- **any** 4 - sided figure
- angles add up to **360°**

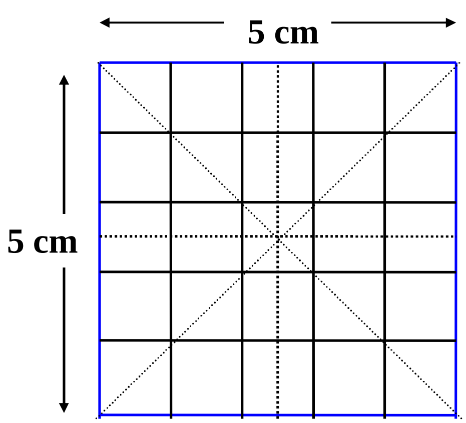


TYPES OF QUADRILATERAL

(i) Square

- 4 **equal** sides
- **90° angle** at **each** corner
- **4 lines** of symmetry (Broken lines on diagram)
- **rotational** symmetry of **Order 4**

Example



Area = 5 cm × 5 cm = 25 cm^2

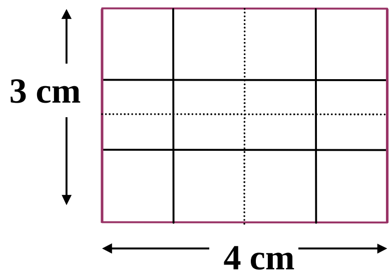
Perimeter = 5 cm × 4 = 20 cm

(ii) Rectangle

- **opposite** sides **equal** and **parallel**
- **90° angle** at **each** corner
- **2 lines** of symmetry

(Broken lines on diagram)

Example



Area = 4cm × 3cm = 12cm²

Perimeter = **4cm + 4cm + 3cm + 3cm = 14cm**

(iii) Parallelogram

- **opposite sides parallel**
- **opposite sides equal**
- **opposite angles equal**
- **diagonals divide each other** in two
- **no line** symmetry
- **rotational symmetry** of **Order 2**

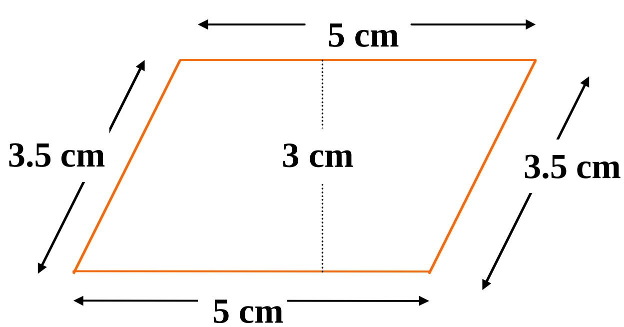
Example

Area = Base × perpendicular height

= 5 cm × 3 cm = 15 cm²

Perimeter = 5 cm + 3.5 cm + 5 cm + 3.5 cm

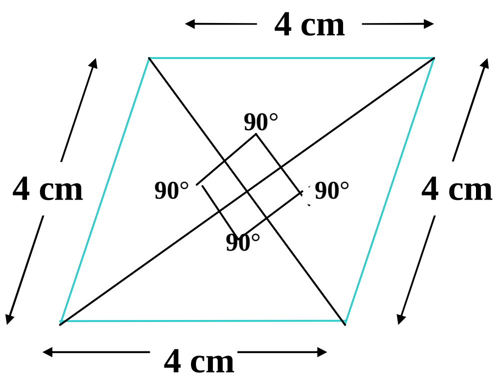
= 17 cm



(iv) **Rhombus**

- a **parallelogram** with **4 equal sides**
- **diagonals meet at right angles** (i.e. 90° angles)
- **2 lines** of symmetry (**each diagonal**)
- **rotational** symmetry of **Order 2**

Example



Area $= \frac{1}{2}$ **product of diagonals**

or

Base ´ perpendicular height

If **long diagonal** = **6.5 cm**

and **short diagonal** = **4.5 cm**

Area $= \frac{1}{2}$ of $6.5\text{ cm} \times 4.5\text{ cm}$

$= \frac{1}{2}$ of 29.25 cm^2

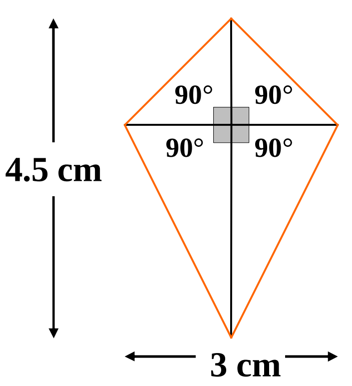
$=$ **14.625 cm**^{2}

(v) **Kite**

- quadrilateral with **2 pairs of equal sides** which are **not usually parallel**, unless for a **rhombus which is also a kite**.

- diagonals **meet at right angles** (90°)
- **long diagonal** is a **line** of symmetry
- **rotational** symmetry of **Order 1**

Example



Area $=$ $\mathbf{\frac{1}{2}}$ **product of diagonals**

$= \frac{1}{2}$ of $4.5\text{ cm} \times 3\text{ cm}$

$= \frac{1}{2}$ of 13.5 **cm**^{2}

$=$ **6.75 cm**^{2}

(vi) **Trapezium** - quadrilateral with **one pair of sides parallel**
- usually **no line** symmetry
- **rotational** symmetry of **Order 1**

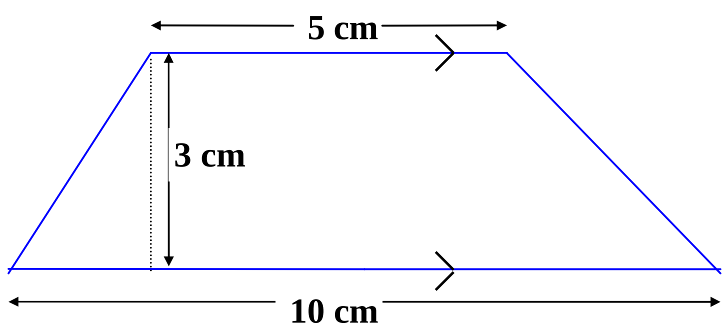
Example

Area $= \frac{1}{2}$ **(Sum of Parallel Sides) ´ perpendicular height**

$= \frac{1}{2}(5 \text{ cm} + 10 \text{ cm}) \times 3 \text{ cm}$

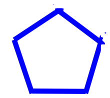
$= 7.5 \text{ cm} \times 3 \text{ cm}.$

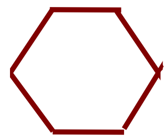
$=$ **22.5 cm** ^{2}

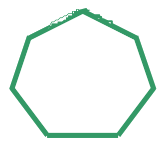


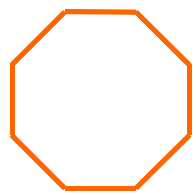
3. SOME OTHER SHAPES

N.B. These shapes, along with Triangles and Quadrilaterals are known as POLYGONS and those with equal sides are REGULAR.

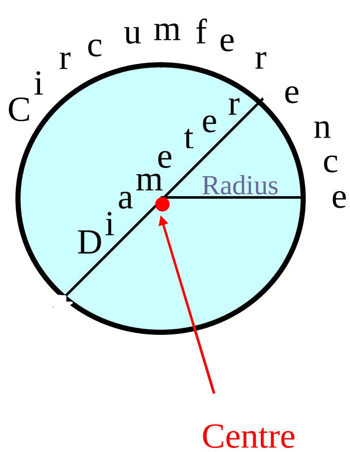
PENTAGON - 5 sides  **Regular Pentagon**

HEXAGON - 6 sides  **Regular Hexagon**

HEPTAGON - 7 sides  **Regular Heptagon**

OCTAGON - 8 sides  **Regular Octagon**

4. CIRCLE

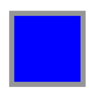
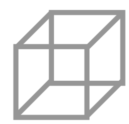


- **diameter cuts** circle into **2 halves**
- **diameter = 2 times radius**
- **angles at centre add up to 360°**
- **each diameter** is a **line** of symmetry
- **rotational** symmetry of **infinite order**
- **circumference** = $\pi \times$ **diameter**
- **area** = $\pi \times$ **radius** ^{2}

(π is approximately **3.14**).

There are **three types of measurement**, namely:

(i) **length** e.g. m, cm, etc.
(ii) **area** e.g. m^2, cm^2, etc.
and
(iii) **volume** e.g. m^3, cm^3, etc.

(i) Length	**(ii) Area**	**(iii) Volume**
		
1 cm	1 cm^2	1 cm^3

Note carefully the difference between the three types of measurement, shown above.

1 cm is just a length and is **one - dimensional measure**.
1 cm^2 is a square which has 1 cm on each edge - the area 1 cm $\times$ 1 cm is **two - dimensional measure**.
1 cm^3 is a cube which has 1 cm on each edge - the **volume** 1cm $\times$ 1 cm $\times$ 1 cm is **three - dimensional measure**.

Then:
1 cm is a length
1 cm^2 is the area of a surface
and **1 cm^3 is the amount of space taken up by the object**.

WORKED EXAMPLE ON THE THREE TYPES OF MEASUREMENT

A length of wire, 36 cm, is bent to form the shape of a cube.
Find:

(a) the length of **each edge** of the cube.

(b) the **area** of material required to **cover the complete frame**.

(c) the **volume** of fine powder required to **fill the cube completely**.

ANSWER

(a) Since a cube has **twelve edges**, the length of each edge in this example is

$$36 \div 12$$

$$= \mathbf{3\ cm}, \text{ which is the answer.}$$

(b) The cube has **six faces**, each a **square** of edge 3 cm.

Area of **1 face** $= 3\text{ cm} \times 3\text{ cm} = 9\text{ cm}^2$

Then, area of **6 faces** is $9\text{ cm}^2 \times 6$

$$= \mathbf{54\ cm^2}, \text{ which is the answer.}$$

(c) **Volume** of Cube:

$$= 3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$$

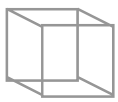
$$= \mathbf{27\ cm^3} \text{ of fine powder, the answer.}$$

NETS

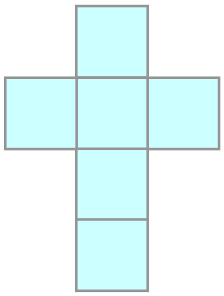
When you are required to find the surface area of a three-dimensional shape, it is often convenient to draw a '**flattened' picture** of the **solid shape** first.

This two-dimensional picture is called a **net** and the **area** of the **net** is the **surface area** of the **solid shape**.

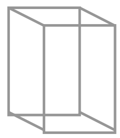
Some examples of nets



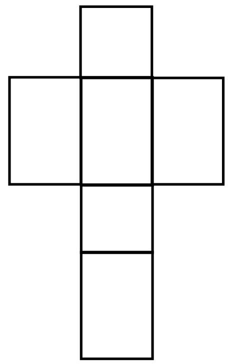
(i) **Cube**



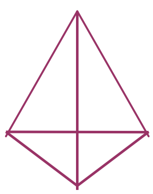
Net of Cube

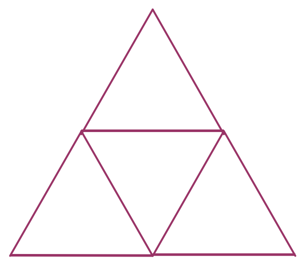


(ii) Cuboid



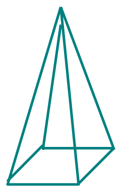
Net of Cuboid

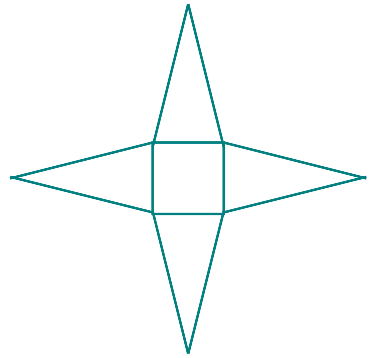




(iii) Triangular - based Pyramid (called a tetrahedron)

Net of a tetrahedron





(iv) Square- based pyramid

Net of square-based pyramid

WORKED EXAMPLE ON A NET

A **cornflakes** packet is a **cuboid**, **30 cm high**, **20 cm long** and **10 cm broad**.

(i) Using a **scale** of **1 mm = 1 cm**, draw a net of the cornflakes packet. Mark the dimensions clearly.

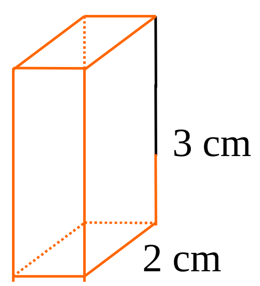
(ii) Use the net to find the area of the surface of the cornflakes packet.

ANSWER

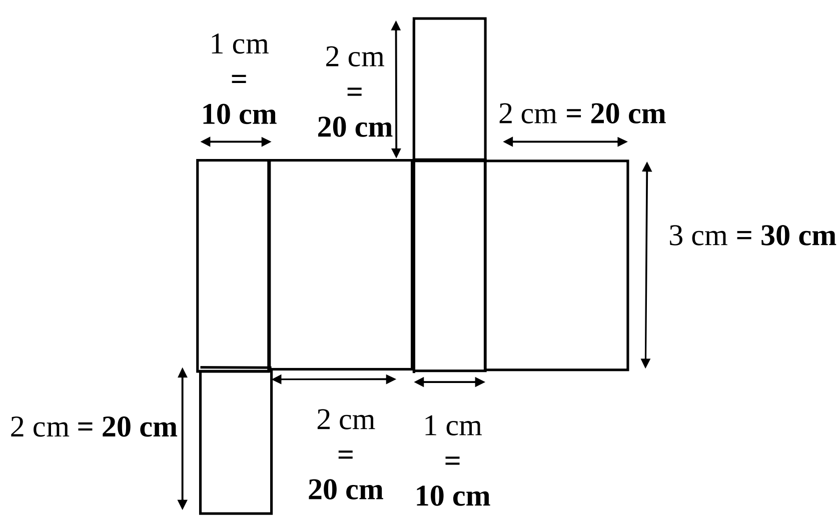
Our scale drawing of the cornflakes packet must have **each of the dimensions divided by 10**, since **10 mm = 1 cm**.

Therefore, our drawing of the packet is **3 cm high**, **2 cm long** and **1 cm broad**.

Our **scale drawing of the packet** looks like this:



(i) Below is a **possible net** of the cornflakes packet.



NOTE: There are **other possible nets** of this packet. You should investigate these for yourself, using a small, empty box.

(ii) Area of net of cornflakes packet:

$$
\begin{aligned}
10 \text{ cm} \times 20 \text{ cm} &= 200 \text{ cm}^2 \times 2 &= 400 \text{ cm}^2 \\
20 \text{ cm} \times 30 \text{ cm} &= 600 \text{ cm}^2 \times 2 &= 1200 \text{ cm}^2 \\
10 \text{ cm} \times 30 \text{ cm} &= 300 \text{ cm}^2 \times 2 &= 600 \text{ cm}^2
\end{aligned}
$$

TOTAL AREA $= \mathbf{2200\ cm^2}$

SYMMETRY

1. Line Symmetry

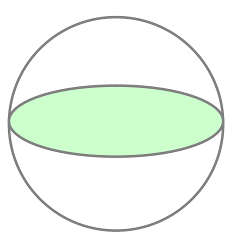
If a shape can be **folded along a line** so that the two portions formed by that line will **fit exactly over each other**, then the line is a **line of symmetry**. A line of symmetry, then, 'cuts' a shape into **two equal parts** , one part being a **mirror image** of the other.

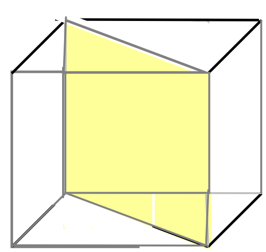
The lines of symmetry are shown as green broken lines on the letters below.

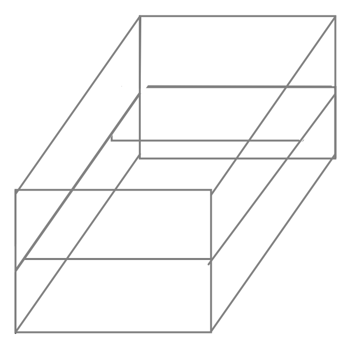
2. Plane Symmetry

If a solid figure such as a sphere, cube or cuboid, is cut into two equal parts, the plane of the cut is known as a **plane of symmetry**. **One plane of symmetry is shaded** in each of the **sphere**, **cube** and **cuboid** shown below:

(i) Sphere **(ii) Cube** **(iii) Cuboid**







(i) The **sphere** has an **infinite number** of planes of symmetry i.e. any cut through the centre.

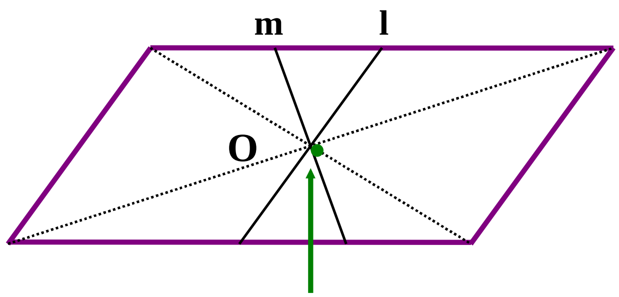
(ii) The **cube** has **9** planes of symmetry. (Check this for your -self using Plasticine.)

(iii) The **cuboid** has **3** planes of symmetry. (Again, check this for yourself.)

3. Point Symmetry

If, in a shape, there is a **point O**, through which **any line** drawn will have the **same distance on either side of O**, then **O** is called a **point of symmetry** and the shape itself is said to have **point symmetry**.

A **parallelogram**, for example, **has point symmetry** since the point where the diagonals meet is a point of symmetry. (i.e. any line drawn through O will have the same distance on either side of O. E.g. lines *l* and *m* below both have **O** as their mid-points).



Point of Symmetry

Notice that the **parallelogram** has **no line symmetry**. Some shapes, e.g. the **square**, **rectangle** and **circle** have **both line** and **point** symmetry.

4. Rotational Symmetry

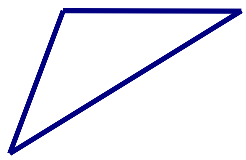
A **rotation** is an **amount of turning** around a **centre** and is **measured in degrees**. A **full rotation**, then, is a full circle, i.e. **360°**.

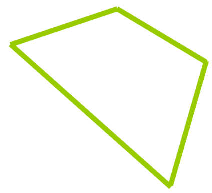
Technically, **every shape** has **rotational** symmetry, since **after a full revolution**, it **will fit exactly over itself** again.

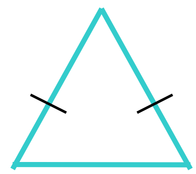
Order of Rotational Symmetry

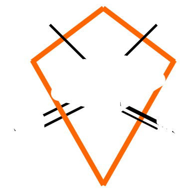
Order 1:
If the shape needs a **full turn** before it will fit exactly over itself again, it is said to have **rotational symmetry of Order 1**.

Examples:


Scalene Triangle

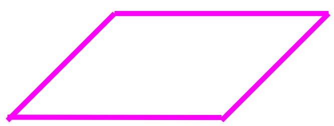

Quadrilateral

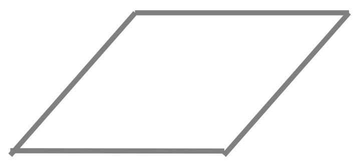

Isosceles Triangle


Kite

Order 2:
If the shape fits exactly over itself after a $\frac{1}{2}$ - **turn**, it has **rotational symmetry of Order 2**.

Examples:

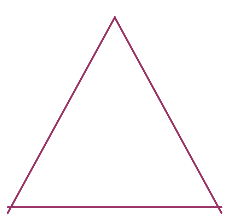

Parallelogram

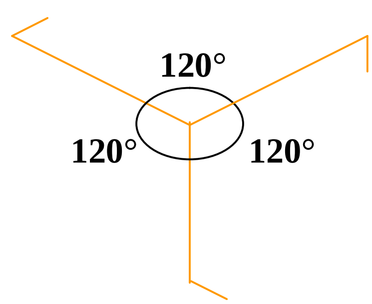

Rhombus


Rectangle

Order 3:
If the shape fits exactly over itself again after a $\frac{1}{3}$ **turn**, **it has rotational symmetry of Order 3.**

Examples:


Equilateral Triangle



Clearly, a **square** has rotational symmetry of **Order 4**
a **regular pentagon** has rotational symmetry of **Order 5**
a **regular hexagon** has rotational symmetry of **Order 6**
and so on.

N.B. The lowest order of rotational symmetry is ONE, since every shape has rotational symmetry. A shape may have NO LINE OR POINT OF SYMMETRY but it ALWAYS has ROTATIONAL SYMMETRY OF AT LEAST ORDER ONE.

EXERCISE 10

(a) Parallelogram
(b) Rhombus
(c) Trapezium
(d) Equilateral Triangle
(e) Isosceles Triangle
(f) Square
(g) Rectangle
(h) Regular Hexagon
(i) Regular Pentagon
(j) Circle
(k) Kite

1. **Use the letters above to answer the following questions**:

(i) The following shapes have at least two lines of symmetry.

(ii) The following shapes have at least two angles equal.

(iii) In the following four-sided shapes, at least one pair of opposite angles are equal.

(iv) In the following four-sided shapes, opposite sides are equal.

(v) In the following four-sided shapes, the diagonals are equal in length.

(vi) In the following four-sided shapes, the diagonals bisect each other.

(vii) In the following four-sided shapes, the diagonals are at right angles to each other.

(viii) The following shapes have at least one pair of sides parallel.

(ix) The following shapes contain at least two right angles.

(x) The following shapes have point symmetry.

(xi) The following shapes have rotational symmetry of, at least, order 2.

2.

Draw each of the shapes (a) to (k) above and **mark** in the following:

(i) Lines of Symmetry (_ _ _ _ _ _ Use broken blue lines)

(ii) Parallel Lines (Mark with green arrows)

(iii) Right Angles (Mark in Red)

(iv) Points of Symmetry (Mark clearly •)

Then, taking each of the shapes **(a)** to **(k)** in turn, write out for **(i)** to **(iv)** how many of each you have marked.

Eg. Shape **(a)** has so many of:

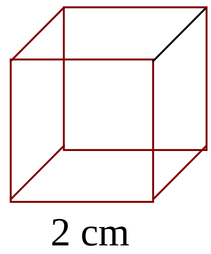
(i) ___ Line (s) of Symmetry
(ii) ___ Pair (s) of Parallel Lines
(iii) ___ Right Angles
(iv) ___ Points of Symmetry

and so on.

3.(**a**) Draw a **net** of each of the following solid shapes:

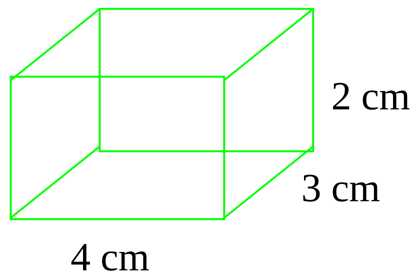
(i)

Cube



(ii)

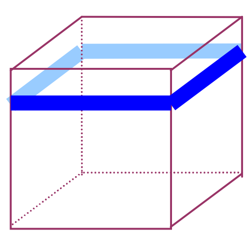
Cuboid



(**b**)(**i**) Use the net for part (**a)(i)** to find the **surface area** of a **cube** of **edge 2 cm**.

(ii) Use the net for part (**a)(ii**) to find the **surface area** of a **cuboid**, **4 cm long**, **3 cm broad** and **2 cm high**.

4.



A Christmas box is in the shape of a **cube** with a length of tape all the way around it as shown. Draw as many nets as you can of this cube.

(**NOTE**: The tape must be drawn correctly on your nets).

EXERCISE 10 - ANSWERS

1. **(i)** (b), (d), (f), (g), (h), (i), (j).

   (ii) (a), (b), (d), (e), (f), (g), (h), (i), (k).

   (iii) (a), (b), (f), (g), (k).

   (iv) (a), (b), (f), (g).

   (v) (f), (g).

   (vi) (a), (b), (f), (g).

   (vii) (b), (f), (k).

   (viii) (a), (b), (c), (f), (g), (h).

   (ix) (f), (g).

   (x) (a), (b), (f), (g), (h), (j).

   (xi) (a), (b), (d), (f), (g), (h), (i), (j).

2.

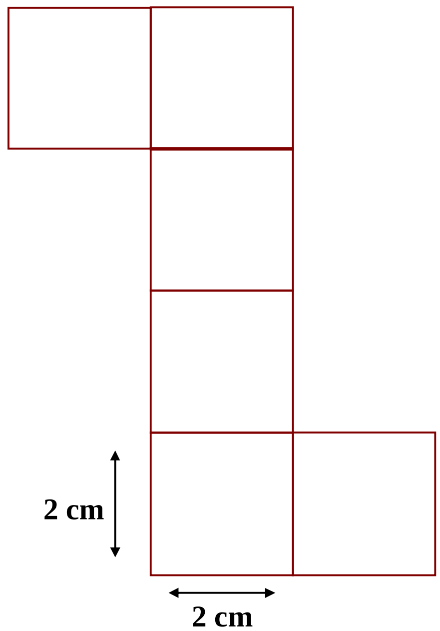
   (i) **(a)** none **(b)** 2 **(c)** 0 **(d)** 3 **(e)** 1 **(f)** 4
   (g) 2 **(h)** 6 **(i)** 5 **(j)** ∞ - Infinite number - any diameter
   (k) 1

   (ii) **(a)** 2 **(b)** 2 **(c)** 1 **(d)** 0 **(e)** 0 **(f)** 2 **(g)** 2
   (h) 3 **(i)** 0 **(j)** 0 **(k)** 0

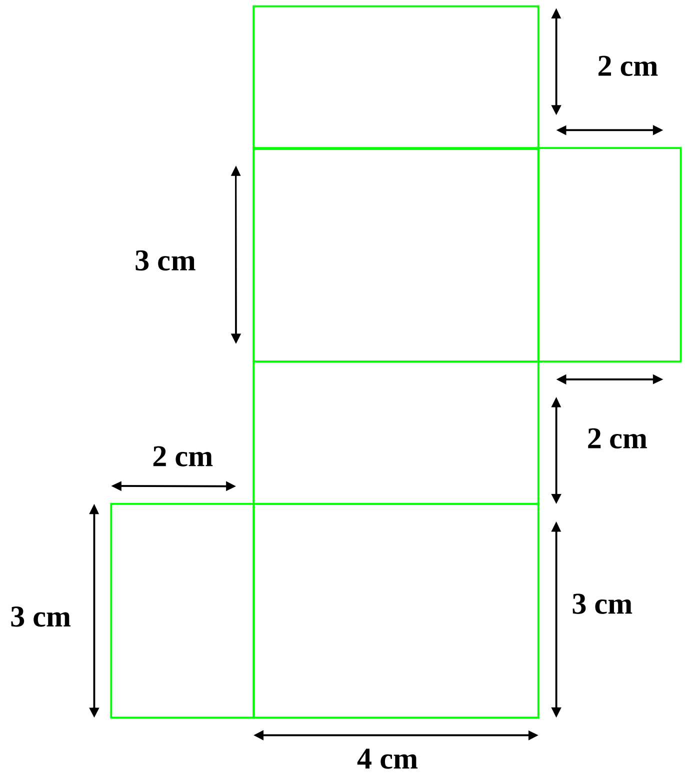
   (iii) **(a)** 0 **(b)** 0 **(c)** 0 **(d)** 0 **(e)** 0 **(f)** 4 **(g)** 4
   (h) 0 **(i)** 0 **(j)** 0 **(k)** 0

   (iv) **(a)** 1 **(b)** 1 **(c)** 0 **(d)** 0 **(e)** 0 **(f)** 1 **(g)** 1
   (h) 1 **(i)** 0 **(j)** 1 **(k)** 0

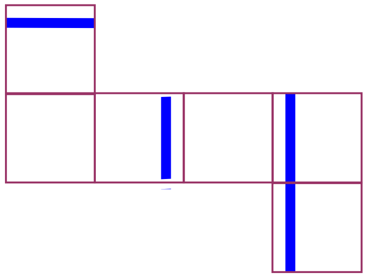
3.**(a) (i)**



(ii)



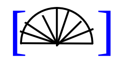
4.



This is one of <u>four</u> possible nets.

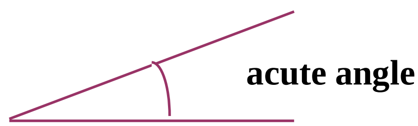
SECTION 11

ANGLES

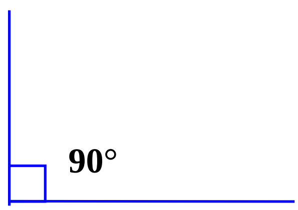
Angles are measured in **degrees**.
An angle of **any** size can be drawn, using a **Protractor**. []

Types of Angle

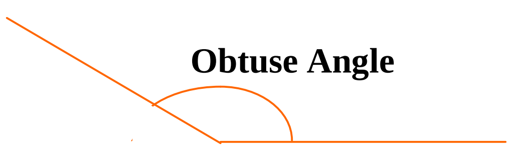
(i) **Acute** angle - less than 90°



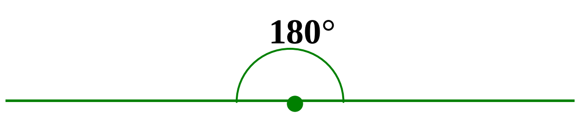
(ii) **Right** angle - 90°



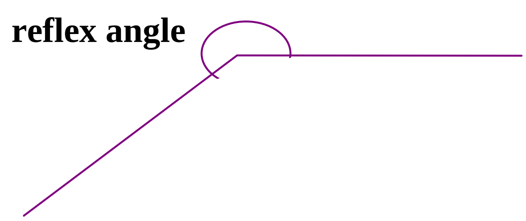
(iii) **Obtuse** angle - over 90° but less than 180°



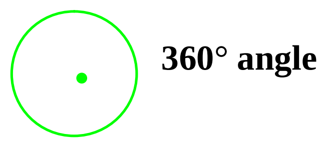
(iv) **Straight Line =** 180°



(v) **Reflex** angle - over 180° but less than 360°

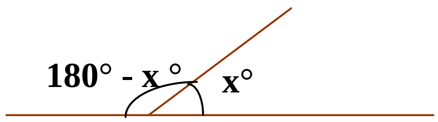


(vi) **Circle -** 360°

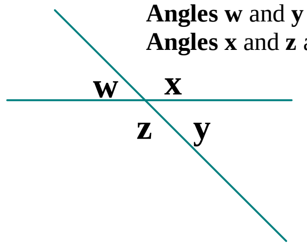


Properties of angles and straight lines

(i) The **total angle** in a straight line is **180°**.



(ii) When **two lines intersect** the **opposite angles** are **equal** to each other; the opposite angles are called **vertically opposite** angles.



Angles w and **y** are **vertically opposite** and, therefore, **equal**.
Angles x and **z** are **vertically opposite** and, therefore, **equal**.

(iii) When **two parallel lines** are cut by a **transversal**:

(a) **corresponding angles** are **equal** **<u>See diagram below</u>**.

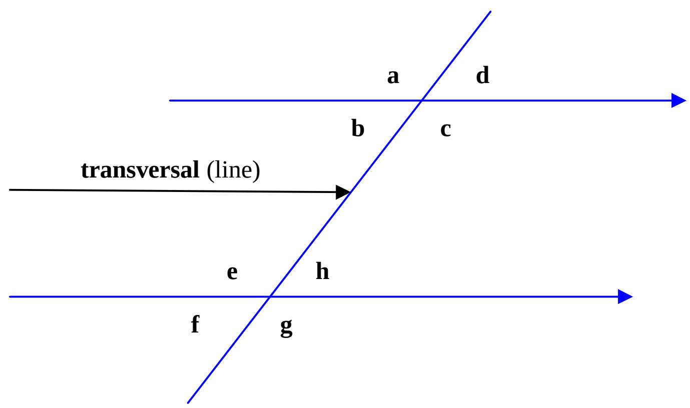
a = e **b = f** **d = h** **c = g**

(b) **alternate angles** are **equal** **<u>See diagram below</u>**.

b = h **c = e**

(c) **interior angles** are **supplementary** (i.e. **add up** to **180°**)

b + e = 180° **c + h = 180°** **<u>See diagram below</u>**.



EXERCISE 11

1. Name the following angles:
Eg. **10°** is **acute**.

(i) 89°	**(ii)** 179°	**(iii)** 205°	**(iv)** 90°
(v) 180°	**(vi)** 270°	**(vii)** 360°	**(viii)** 8°
(ix) 181°	**(x)** 359°		

2. **Draw** each of the following angles **using a protractor**:

(i) 15°	**(ii)** 165°	**(iii)** 84°	**(iv)** 200°
(v) 19°	**(vi)** 10°	**(vii)** 89°	**(viii)** 98°
(ix) 120°	**(x)** 60°		

3. **Using your protractor**, **draw a triangle** for each of the following, **using the two angles given** in each case:

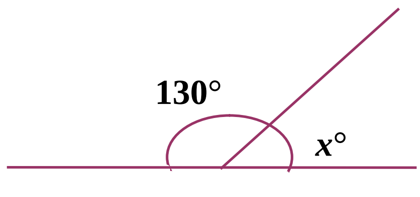
(i) 20°, 60°	**(ii)** 100°, 50°	**(iii)** 75°, 75°	**(iv)** 21°, 89°
(v) 62°, 51°	**(vi)** 80°, 90°	**(vii)** 130°, 40°	**(viii)** 18°, 19°
(ix) 60°, 100°	**(x)** 10°, 60°		

4. **(a)** **Using your protractor**, **measure the third angle** in each of the 10 triangles you have drawn for No.3.

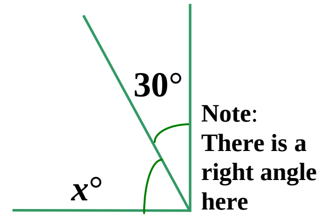
(b) What do you notice about the **sum of the 3 angles in each triangle**?

5. **Calculate** the size of the angle marked $x°$ in each of the following:
(NOTE: No need for a protractor - these are not drawn accurately)

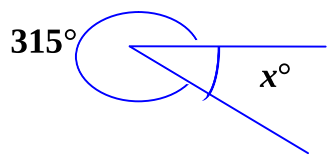
(i)



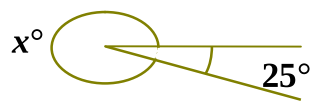
(ii)



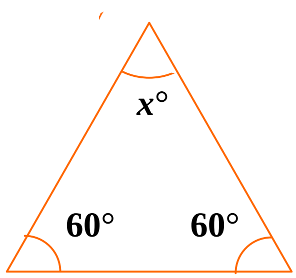
(iii)



(iv)

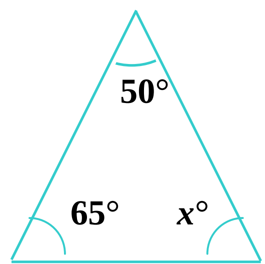


(v)



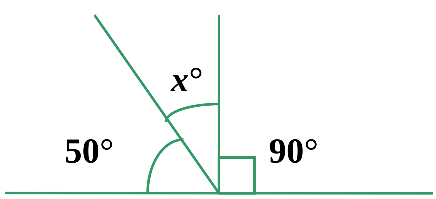
What type of triangle is this ?

(vi)

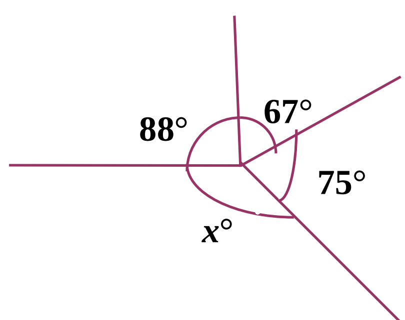


What type of triangle is this ?

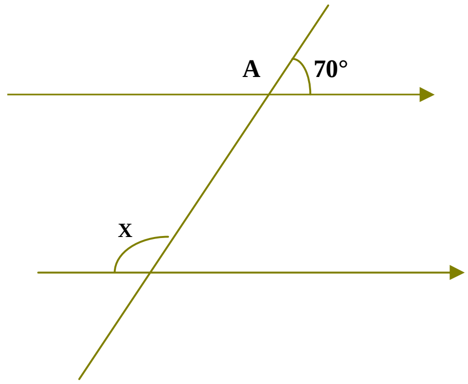
(vii)



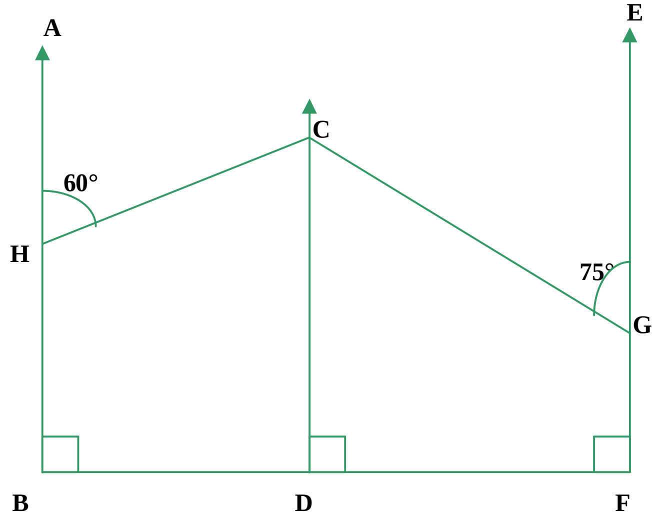
(viii)



6. Find the angle **x** shown in the figure below.
Show your working, giving a **reason** for each statement.



7. Find the angle **HCG** in the figure below.
Show your working, giving a **reason** for each statement.

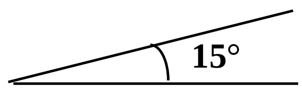


EXERCISE 11 - ANSWERS

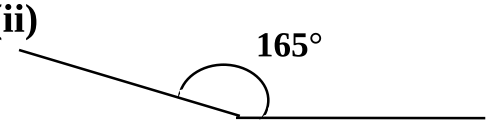
1. **(i)** Acute **(ii)** Obtuse **(iii)** Reflex **(iv)** Right **(v)** Straight Line
(vi) Reflex **(vii)** Circle **(viii)** Acute **(ix)** Reflex **(x)** Reflex

2. **(**Your answers should **resemble** the following angles.)
NOTE: The following are for guidance purposes only and are **not drawn accurately.**

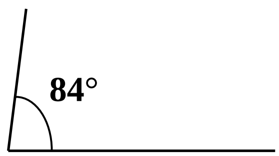
(i)



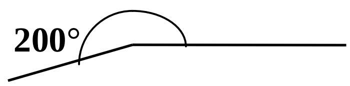
(ii)



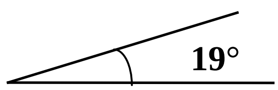
(iii)



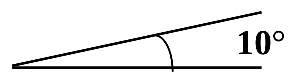
(iv)



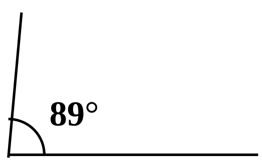
(v)



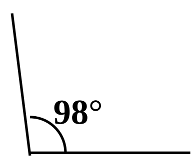
(vi)



(vii)

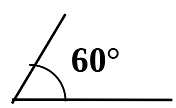


(viii)



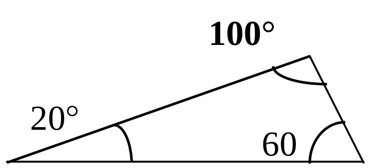
(ix)

(x)

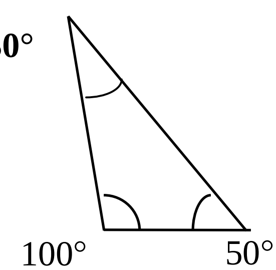


3. Your answers should **resemble** the following triangles.
NOTE : Again, these are for guidance purposes only and are **not drawn accurately.**

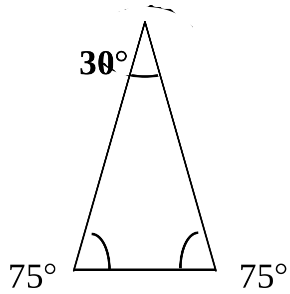
(i)



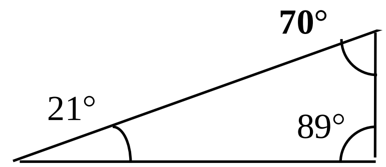
(ii)



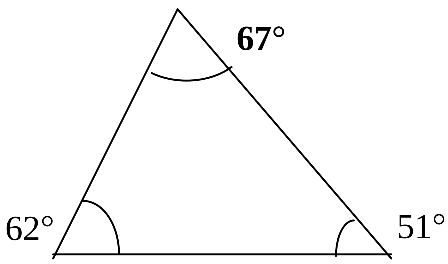
(iii)



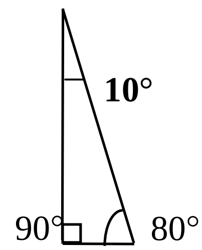
(iv)



(v)



(vi)



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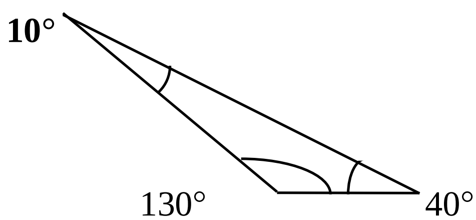
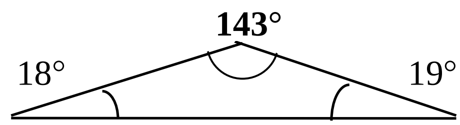
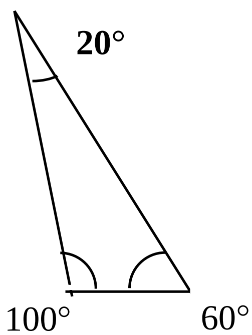


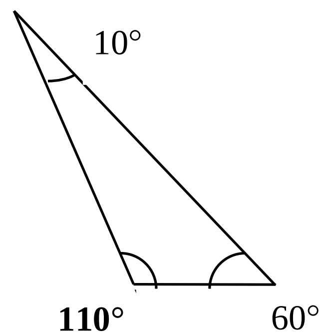
(viii)



(ix)



(x)



4.**(a)** **(i)** 100° **(ii)** 30° **(iii)** 30° **(iv)** 70° **(v)** 67° **(vi)** 10°
(vii) 10° **(viii)** 143° **(ix)** 20° **(x)** 110°

(b) It is always 180°.

5. **(i)** 50° **(ii)** 60° **(iii)** 45° **(iv)** 335° **(v)** 60° ; equilateral
(vi) 65° ; isosceles **(vii)** 40° **(viii)** 130°

6. Angle **A = (180 – 70)° = 110°** (Angles in **straight line** add up to **180°**)

Angle **x = Angle A** (**Corresponding angles equal**)

\ Angle **x = 110°**.

7. The line **BF** is a **transversal** through the lines **AB**, **CD** and **EF**.

Angle **ABD =** Angle **CDF = 90°** (Given)
\ Angles **ABD** and **CDF** must be **corresponding angles**
\ **AB** and **CD** are **parallel** to each other.
Angle **AHC =** Angle **HCD = 60°** (**Alternate angles equal**).

Angle **EFD =** Angle **CDF = 90°** (Given)
\ Angles **EFD** and **CDF** must be **interior angles** (They add up to 180°)
\ **EF** and **CD** are **parallel** to each other.
Angle **DCG =** Angle **CGE = 75°** (**Alternate angles equal**).

Angle **HCG =** Angle **HCD +** Angle **DCG**
\ Angle **HCG = 60° + 75° = 135°**.

SECTION 12

ALGEBRA – INTRODUCTION

What is Algebra?

The word **'algebra'** comes from the Arabic word **'algorithm'**, meaning **'a step-by-step process for performing calculations'.** **Algebra** is a **special kind of arithmetic which uses letters (or symbols)** instead of numbers to represent quantities. The only difference is that ***x***, for example, can stand for **any quantity**, whereas a **number** like **3**, for example, **stands only** for a **set of three things**.

In calculations, ***x*** is used in exactly the same way as **3**, or **any** other number.

THE FOUR BASIC RULES

The **four basic rules**, namely:

addition, subtraction, multiplication and **division**,

are applied in **algebra** in the **same way** as they are in **arithmetic**.

(i) Addition

$x + 3$ means **3** is **added to** ***x***.
$x + 3x$ means **1 of** ***x*** is added to **3 of** ***x***, giving **4 of** ***x*** altogether;
we call this $4x$.
Note the difference between $x + 3$ and $x + 3x$.
$x + y$ means a quantity ***x*** is added to a quantity ***y***.
Note that $x + y$ is the **same** as $y + x$;
since the **order** is **not important** we say that quantities are **commutative under addition.**

(ii) Subtraction

3 - ***x*** means ***x*** **is subtracted from 3**.

3*x* **-** ***x*** means **1 of** ***x*** is subtracted from **3 of** ***x***,
leaving **2 of** ***x*.** We call this **2*x***.

N.B Note the **difference** between **3 -** ***x*** and **3*x*** **-** ***x***.

Also: ***x*** **-** ***y*** is **not the same** as ***y* – *x*.**

Since the **order is important**, we say that quantities **are not commutative under substraction.**

(iii) Multiplication

There is **no need to use a multiplication sign (**$\times$**)** in algebra:

2*x* means **2** $\times$ ***x*** or ***x*** **+** ***x*** and
5*x* means **5** $\times$ ***x*** or ***x*** **+** ***x*** **+** ***x*** **+** ***x*** **+** ***x***

As in arithmetic, **multiplication is a short method of addition**, where we have:

2 ´ **9** short for **9 + 9** and

5 ´ **9** short for **9 + 9 + 9 + 9 + 9.**

xy means a quantity ***x*** is multiplied by another quantity ***y***.
Note that ***xy*** is the **same** as ***yx***.

Since the **order is not important**, we say that quantities are **commutative under multiplication**.

(iv) Division

$\frac{x}{3}$ means ***x*** is divided by **3**, giving $\frac{1}{3}$ of ***x*.**

$\frac{x+1}{3}$ means **1 is added to** ***x*** and this **result** is **divided by** ***y*** **3**.

$\frac{x}{y}$ means ***x*** is divided by ***y*** and

$\frac{y}{x}$ means ***y*** is divided by ***x***.

Since $\frac{x}{y}$ is **not the same** as $\frac{y}{x}$, as, for instance, **4** ¸ **2** is **not equal** to **2** ¸ **4**, we say that quantities are **not commutative under division**.

BRACKETS

Brackets are used like a **pocket** to hold things safely together. The **contents** of brackets must be treated as a **single quantity** and **worked out on their own**:

Eg. (i) **3(*x* + 1)** means **1 is added to *x*** and this **result** is **multiplied by 3**.

Eg. (ii) **2(3*x* - 1)** means **1 is subtracted from 3 times *x*** and **the result** is **doubled**.

It is possible to **remove brackets** by **multiplying each term inside** the brackets by the **'multiplier' outside** the **brackets**.

Removing brackets:

Eg. **(i)** **3(*x* + 1)** is **equal** to **3*x* + 3**

and **(ii)** **2(3*x* – 1)** is equal to **6*x* – 2**.

POWERS

x can be raised to a **power** which means it is **multiplied by itself** the number of times that the power states:

x^2 means $x \times x$.

Note the **difference** between x^2 and $2x$:

$$x^2 = x \times x \quad \text{but} \quad 2x = x + x.$$

If $x = 4$, then $x^2 = 4 \times 4 = 16$

but $2x = 2 \times 4 = 8$.

Again, note the **difference** between x^3 and $3x$:

$$x^3 = x \times x \times x \text{ but } 3x = 3 \times x.$$

If $x = 4$, then $x^3 = 4 \times 4 \times 4 = 64$

and $3x = 3 \times 4 = 12$.

LIKE TERMS

Since all terms containing x**,** for example**,** in any algebraic expression are **the same**, they can be **collected together** to form a **single term** in x, by **adding, subtracting, multiplying** and **dividing** as required. The **terms** that are **alike** are called **like terms** and adding and subtracting them to form a **single term** in x is **collating like terms**.

> To **express** an **algebraic** expression in its **simplest** form, **like terms must be collated**.

Eg.(i) **Simplify** $2x - 3x + 5x - 4x + x + 1 - 5 + 2$.

Collating like terms, we have:

$$+2x - 3x + 5x - 4x + 1x = +1x = x \text{ and}$$
$$+1 - 5 + 2 = -2,$$

giving $x - 2$ as the **simplest form**.

Like Terms (contd.)

Eg.(ii) **Simplify** $2x + 1 + x - 3 - 4x + 5 - x - 10$.

Collating like terms, we have:

$+2x + 1x - 4x - 1x \quad = -2x$

and $\quad +1 - 3 + 5 \quad = +3$,

giving $\quad -2x + 3 \quad$ as the **simplest form**.

Eg.(iii) **Simplify** $-xy + x - 2xy + 3 - 4x - 7 + z - 5z + p$.

Collating like terms, we have:

$-1xy - 2xy = -3xy$,

$+1x - 4x = -3x$,

$+3 - 7 = -4$

and $\quad +1p = +p$,

giving $\quad -3xy - 3x - 4 + p \quad$ as the **simplest form**.

Eg. (iv) Write the following in its **simplest form**, by **firstly removing** the **brackets**, and then **collating like terms**:

$2x - 7 - (3x + 5) + 3y - 2 + 5(2y - 2) + (y - 4) + 10$

N.B. $-(3x + 5)$ is $\underline{\mathbf{-1}(3x + 5)}$ and $+(y - 4)$ is $\underline{\mathbf{+1}(y - 4)}$.

(Worked Answer on next page).

Eg. (iv): **Removing brackets** gives:
$\mathbf{2x - 7 - 3x - 5 + 3y - 2 + 10y - 10 + y - 4 + 10}$

Collating *x* terms, we have:
$\mathbf{2x - 3x = -1x = -x}$.

Collating *y* terms we have:
$\mathbf{+3y + 10y + 1y = +14y}$.

Collating numbers, we have:
$\mathbf{-7 - 5 - 2 - 10 - 4 + 10 = -18}$

The **simplest form** of the **whole expression** is, therefore:

$$\mathbf{-x + 14y - 18.}$$

EXERCISE 12(A)

(Algebra - the four rules, use of symbols, removing brackets and simplifying.)

1. Write the following **English** in the 'language' of **algebra**:
   - (i) A quantity ***x*** is **added** to **two**.
   - (ii) A quantity ***y*** is **subtracted** from **five**.
   - (iii) **Twice** a quantity ***x*** is **added** to **three**.
   - (iv) **Thirteen** is **subtracted** from **four times** ***x***.

2. Write the following **algebra** in the language of **English**:
   - **(i)** $\mathbf{4 + x}$.
   - **(ii)** $\mathbf{6 - x}$.
   - **(iii)** $\mathbf{5x - 1}$.
   - **(iv)** $\mathbf{12 - 7x}$.

3. Write the following **English** in **algebra**:
   - (i) A quantity ***x*** is divided by **two**.
   - (ii) **Two** is divided by a quantity ***y***.
   - (iii) **One** is added to ***x*** and the **result** is **doubled**.
   - (iv) **One** is added to ***x*** and the **result** is **halved**.

4. Write the following **algebra** in **English**:

(i) $\frac{x}{2}$

(ii) $\frac{2}{x}$

(iii) $\frac{x+1}{2}$

(iv) $\frac{x+y}{x-y}$

5. **Remove the brackets** from each of the following:

(i) $2(x-4)$.

(ii) $3(2x+6)$.

(iii) $-4(5x+4)$.

(iv) $-(2x+1)$. (**N.B.** This means $-1(2x+1)$.)

6. **Simplify** each of the following, by **removing brackets** and then **collating like terms**:

(i) $2(x+4)-3x-9$.

(ii) $6x-3(2x+1)+6$.

(iii) $-(2x+1)+2x+1$.

(iv) $3(2x-2)+2(3x-2)$.

(v) $2(x-1)-3x+(3x-1)$.

(vi) $x(2x-3)+x^2(x+y)-7x+3$.

PRACTICAL APPLICATIONS OF ALGEBRA STUDIED EARLIER

1. A **rectangular** floor has its **length 3 times** its **width**. If the **width** is ***x* metres**, find in terms of ***x***:

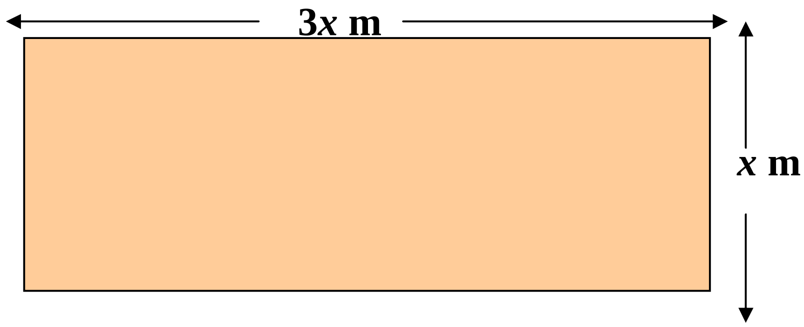
(a) the **length**,

(b) the **perimeter**

and **(c)** the **area**,

in their **simplest form**.

The “picture” looks like this:



(a) The **length** is **3 times *x*,** which is **3*x* m**.

(b) The **perimeter** is ***x* + 3*x* +*x* + 3*x* = 8*x* m,**

in its **simplest form**.

(c) The **area** is **length ´ width**:

= 3*x* ´ *x*
= $3x^2$ m ^{2},

in its **simplest form**.

2. A **rectangle** has its **length 3m more** than its **width**.

If the **width** is ***x* m**, find, in terms of ***x***:

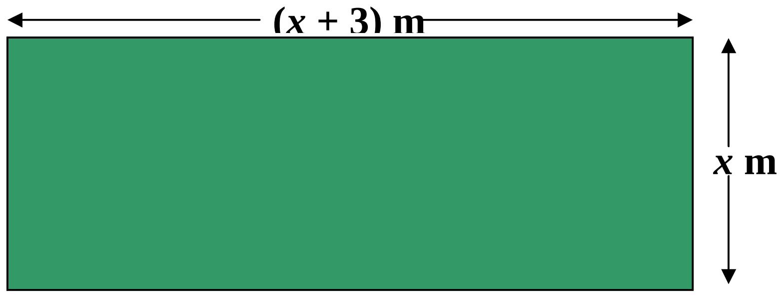
(a) the length,

(b) the perimeter

and **(c)** the area,

in their **simplest form**.

The “picture” looks like this:



(a) This time, the **length** is **3m more** than ***x***, which is:

(***x*** **+ 3)m**.

(b) The **perimeter** is: ($\boldsymbol{x + 3 + x + x + 3 + x}$) **m**

= **(4*x* + 6)m** or **2(2*x* + 3)m**,

in its **simplest form**.

(c) The **area**, **length** ´ **width**, is:

$\boldsymbol{x(x + 3)\text{m}^2}$ or ($\boldsymbol{x^2 + 3x}$) $\boldsymbol{\text{m}^2}$,

in its **simplest form**.

EXERCISE 12(B)

(Some Practical Applications of Algebra)

(NOTE: When answering a question involving a shape, it is advisable to draw a diagram.)

1. A **rectangular** garden has its **length four times** its **width**.
   If the **width** is ***x* m**, find, in terms of ***x***:

   (a) the **length**,
   (b) the **perimeter** and
   (c) the **area**, in their **simplest form**.

2. A **rectangle** has its **length 4 cm longer** than its **width**.
   If the **width** is ***y* cm**, find, in terms of ***y***:
   (a) the **length**,
   (b) the **perimeter** and
   (c) the **area**,
   in their **simplest** form.

EXERCISE 12(A) - ANSWERS

1 **(i)** $x + 2$ **(ii)** $5 - y$ **(iii)** $2x + 3$
(v) $4x - 13$

2. **(i)** A quantity ***x*** is **added** to **four**.
(ii) A quantity ***x*** is **subtracted** from **6**.
(iii) **One** is **subtracted** from **five times *x***
(iv) **Seven times *x*** is **subtracted** from **twelve**.

3. **(i)** $\frac{x}{2}$ **(ii)** $\frac{2}{y}$ **(iii)** $2(x + 1)$

(iv) $\frac{x+1}{2}$ **or** $\frac{1}{2}(x + 1)$

4. **(i)** A quantity ***x*** is **divided** by **2**.

(ii) **2** is **divided** by a quantity ***x***.

(iii) **1** is **added** to ***x*** and the **result** is **halved**.

(iv) The **sum** ***x+y*** is **divided** by the **difference** ***x-y***,
for two quantities ***x*** and ***y***.

5. **(i)** $2x - 8$ **(ii)** $6x + 18$ **(iii)** $-20x - 16$
(v) $-2x - 1$

6. ***Brackets Removed*** ***Like Terms Collated (i.e expression simplified)***

(i) $2x + 8 - 3x - 9$ $-x - 1$

(ii) $6x - 6x - 3 + 6$ 3

(iii) $-2x - 1 + 2x + 1$ 0

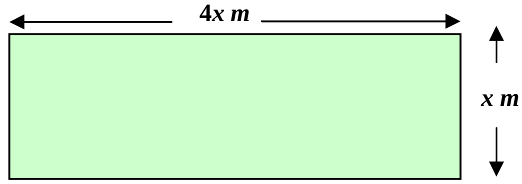
(iv) $6x - 6 + 6x - 4$ $12x - 10$ **or** $2(6x - 5)$

(v) $2x - 2 - 3x + 3x - 1$ $2x - 3$

(vi) $2x^2 - 3x + x^3 + x^2y - 7x + 3 = 2x^2 - 10x + x^3 + x^2y + 3$

EXERCISE 12(B) - ANSWERS

1. The "picture" looks like this:

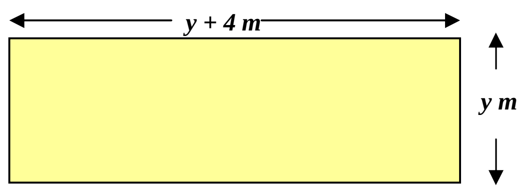


(a) $\mathbf{4x}$ **m**.

(b) $\mathbf{x + 4x + x + 4x \quad = \quad 10x}$ **m**.

(c) $\mathbf{4x \times x = 4x^2}$ **cm^2**.

2. The "picture" looks like this:



(a) $\mathbf{(y + 4)}$ **cm**.

(b) $\mathbf{y + y + 4 + y + y + 4 \quad = \quad (4y + 8)}$**cm** or $\mathbf{4(y + 2)}$ **cm**.

(c) $\mathbf{y(y + 4)}$ **cm^2** or $\mathbf{(y^2 + 4y)}$ **cm^2**.

SECTION 13

SIMPLE EQUATIONS

Simple Equations

The word **'equation'** means **'equality'**.

An equation is simply a statement that **two quantities** are **equal**, that is:

Left-hand Side = *Right-hand Side*

They would '**balance** ' if '**weighed on a pair of scales**'.

Examples of arithmetic equations :-

(i) $\mathbf{11 + 4 - 3 = 6 \times 2}$
Left-hand side = **12** and Right -hand side = **12**.

(ii) $\mathbf{2 \times (9-1) = 4 \times 4}$
Left-hand side = **16** and Right -hand side = **16**.

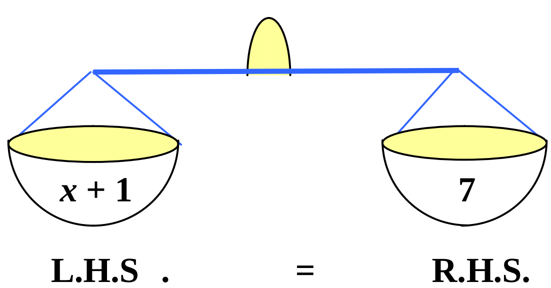
(iii) $\mathbf{2 + (7 \times 3) - 23 = 0}$
Left-hand side = **0** and Right -hand side = **0**.

Now we shall look at some **algebraic** equations
and **find the unknown quantity** in each:

Examples of algebraic equations :-

(i) $x + 1 = 7$

If we put these quantities on the 'scales' they would **balance**:



(Solution on next page).

Subtract 1 from each side:

$$\begin{array}{rcl} x + 1 & = & 7 \\ \underline{\quad -1} & & \underline{-1} \\ x & = & 6. \end{array}$$

We have just **solved the equation**:

$$x + 1 = 7,$$

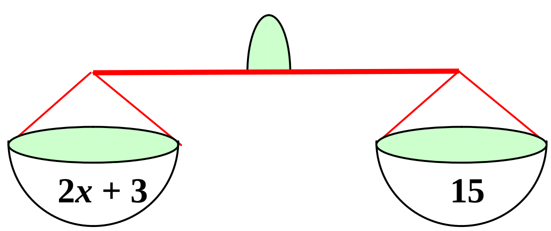
and found the **solution** to be:

$$x = 6.$$

(Check: **6** + 1 = 7).

(ii) $2x + 3 = 15$

Again, these quantities would **balance** on the scales :



$$2x + 3 = 15$$

***Subtract 3 from each side*:**

$$\begin{array}{rcl} \underline{-\ 3} & & \underline{-3} \\ 2x & = & 12 \end{array}$$

***Divide each side by 2* :** $x = 6.$

We have now **solved the equation**:

$$2x + 3 = 15$$

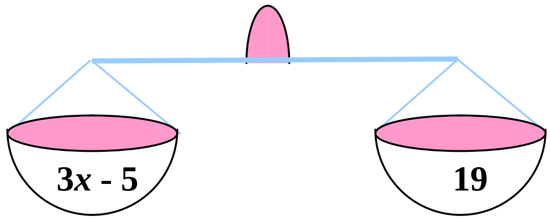
and found **the solution** to be:

$$x = 6.$$

(Check: $2 \times \mathbf{6} + 3 = 15$).

(iii) Solve $\quad \mathbf{3x - 5 = 19}$

Putting these on the scales, we have :



$$\begin{array}{lrcl}
 & 3x - 5 & = & 19 \\
\textit{Add 5 to each side}: & \underline{+\,5} & = & \underline{+\,5} \\
 & 3x & = & 24 \\
\textit{Divide each side by 3}: & x & = & 8\ .
\end{array}$$

(N.B. We must **add 5** to **-5** to get rid of it**)**.

(Check: $3 \times \mathbf{8} - 5 = 19$).

(iv) Solve $\quad \mathbf{2x + 1 = x + 5}$

$$\begin{array}{lrcl}
 & 2x + 1 & = & x + 5 \\
\textit{Subtract 1 from each side}: & \underline{-1} & & \underline{-1} \\
 & 2x & = & x + 4 \\
\textit{Subtract x from each side}: & \underline{-x} & & \underline{-x} \\
 & x & = & 4
\end{array}$$

(Check: $2 \times \mathbf{4} = \mathbf{4} + 4$,
so, $\mathbf{x = 4}$ is the **solution**).

(v) Solve $\quad \mathbf{3x - 2 = x + 8}$

$$\begin{array}{lrcl}
\textit{Add 2 to each side}: & \underline{+2} & & \underline{+\,2} \\
 & 3x & = & x + 10 \\
\textit{Subtract x from each side}: & \underline{-x} & & \underline{-x} \\
 & 2x & = & 10 \\
\textit{Divide each side by 2}: & x & = & 5.
\end{array}$$

so $\mathbf{x = 5}$ is the **solution**.

(Check: $3 \times \mathbf{5} - 2 = \mathbf{5} + 8$).

(vi) Solve $\quad \mathbf{4x - 11 = 2x - 5}$

$$
\begin{array}{lrcr}
\textit{Add 11 to each side:} & \underline{\mathbf{+11}} & & \underline{\mathbf{+11}} \\
 & \mathbf{4x} & = & \mathbf{2x + 6} \\
\textit{Subtract 2x from each side:} & \underline{\mathbf{-2x}} & = & \underline{\mathbf{-2x}} \\
 & \mathbf{2x} & = & \mathbf{6} \\
\textit{Divide each side by 2:} & \mathbf{x} & = & \mathbf{3}.
\end{array}
$$

so $\boldsymbol{x} = \mathbf{3}$ is the **solution**.

(Check: $4 \times \mathbf{3} - 11 = 2 \times \mathbf{3} - 5$).

Construction of Simple Equations

Sometimes we are faced with a mathematical problem that is difficult to solve by arithmetical methods.
If we **represent the unknown quantity by a letter** or a symbol, we can **construct an equation** to suit the data of the problem.
The solution to this equation gives the unknown quantity.

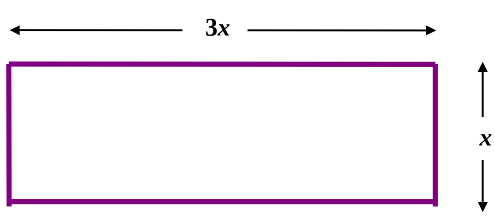
Eg. (i):

If the **length** of a room is **three times** the **width** and the **perimeter** is **14m**, find:

(a) the **width** of the room
and
(b) the **length** of the room.

Let $\boldsymbol{x}$ be the **width**.
Then $\mathbf{3x}$ is the **length**.

The **diagram** looks like this:



The **total perimeter** is:

$$\boldsymbol{x + 3x + x + 3x} \quad = \quad = \mathbf{8x}.$$

Eg.(i) **(Contd.)**

Then the **equation** is: $\mathbf{8x = 14}$

Divide each side by 8 : $\mathbf{x = 1.75m}$

and $\mathbf{1.75m \times 3 = 3x = 5.25\ m}$

Answers:

(a) Width = 1.75 m.

(b) Length = 5.25 m.

Eg. (ii):

The **sum** of **three consecutive whole numbers** is **204**.
Find the three numbers.

Let $\boldsymbol{x}$ be the **smallest** number.
Then $\boldsymbol{x+1}$ is the **second** number
and $\boldsymbol{x+2}$ is the **third** number.
The **sum** of these numbers is :

$$\boldsymbol{x + x + 1 + x + 2}$$
$$= \mathbf{3x + 3}.$$

Then **the equation** is $\mathbf{3x + 3 = 204}$

Subtract 3 from each side:

$$\begin{aligned} 3x + 3 &= 204 \\ \underline{-3} &\quad \underline{-3} \\ 3x &= 201 \end{aligned}$$

Divide each side by 3: $\boldsymbol{x} = \mathbf{67}$.

Then $\boldsymbol{x+1} = \mathbf{68}$

and $\boldsymbol{x+2} = \mathbf{69}$.

Therefore, the **three consecutive numbers** are :

67, 68 and 69.

Eg. (iii):

A **rectangular** room has its **length 3m longer** than its **width**. If the **total perimeter** is **22m** find :

(a) The **length** and **width** of the room

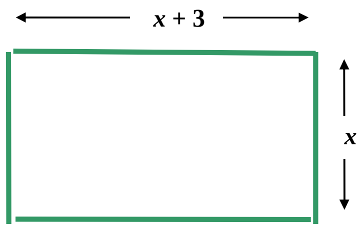
and

(b) the **area** of the room.

Eg.(iii) ***(Contd.)***

(a) Let $\boldsymbol{x}$ = **width**.
Then $\boldsymbol{x + 3}$ = **length**.

The **diagram** looks like this:



The **total perimeter** is :

$\boldsymbol{x + x + 3 + x + x + 3} \quad = \quad \boldsymbol{4x + 6}$

Then the equation is: $\quad \boldsymbol{4x + 6 = 22}$

Subtract 6 from each side: $\quad \underline{\boldsymbol{-6}} \quad \underline{\boldsymbol{-6}}$

$\boldsymbol{4x = 16}$

Divide each side by 4: $\quad \boldsymbol{x = 4}$

and $\quad \boldsymbol{x + 3 = 7}$.

Therefore, the **length** of the room is **7m** and the **width** of the room is **4m**.

(b) Area of floor = length × width

$= \mathbf{7m \times 4m}$

$= \mathbf{28m^2}$.

EXERCISE 13

(NOTE: When answering a question involving a shape, it is advisable to draw a diagram.)

(Construction and Solution of Simple Equations)

1. **Solve** the following equations:

(a) $x + 2 = 13$
(b) $x - 2 = 13$
(c) $2x + 3 = 11$
(d) $2x - 3 = 11$
(e) $2x + 1 = x + 4$
(f) $2x - 1 = x - 4$
(g) $3x - 1 = 4x + 3$
(h) $4x - 1 = 2x + 7$
(i) $5x + 2 = 2x - 7$
(j) $6x + 5 = 5x + 6$

2. If the **length** of a **rectangular** floor is **twice** its **width** and the **perimeter** of the floor is **18m**, find:
(a) the **width**,
(b) the **length** and
(c) the **area**
of the floor.

3. A **rectangle** has its **length 2cm longer** than its **width** and the **perimeter** is **40cm**. Find:
(a) the **width**,
(b) the **length** and
(c) the **area**
of the rectangle.

4. The **sum** of **four consecutive numbers** is **66**.
Find the numbers.

EXERCISE 13 - ANSWERS

1. **(a)** **11** **(b)** **15** **(c)** **4** **(d)** **7** **(e)** **3**

   (f) **-3** **(g)** **-4** **(h)** **4** **(i)** **-3** **(j)** **1**.

2. Let $\boldsymbol{x}$ be the **width**; then $\mathbf{2}\boldsymbol{x}$ is the **length**.

   The **perimeter** is, therefore:

   $(x + 2x + x + 2x)$ **m** = $6x$ **m.**
   So $6x = 18$,
   giving $x = 3$ and $2x = 6$.

   (a) **3m** **(b)** **6m** **(c)** $\mathbf{18m^2}$.

3. Let $\boldsymbol{w}$ be the **width**; then $\boldsymbol{w} + \mathbf{2}$ is the **length**.

   The **perimeter** is, therefore:
   $(w + w + 2 + w + w + 2)$ **cm** = $(4w + 4)$**cm** or $4(w + 1)$**cm**.

   So $4w + 4 = 40$,
   giving $w = 9$ and $w + 2 = 11$.

   (a) **9cm**. **(b)** **11cm**. **(c)** $\mathbf{99cm^2}$.

4. Let $\boldsymbol{x}$ be the **smallest** of the four numbers.

   Then $x + 1$ is the next smallest,
   $x + 2$ is next
   and $x + 3$ is the **greatest**.

   The **sum** of these numbers is, therefore:
   $x + x + 1 + x + 2 + x + 3 = 4x + 6$ or $2(x + 3)$.

   So $4x + 6 = 66$
   giving $x = 15$.

   Then $x + 1 = 16$,
   $x + 2 = 17$
   and $x + 3 = 18$.

SECTION 14

SIMPLE INEQUATIONS & THE NUMBER LINE

Simple Inequations

The word **'inequation'** means **'inequality'.**
It is a statement, then, that **two quantities are unequal,** that is **one is greater than the other** or **one is less than the other**.

Examples of arithmetic inequations:

(i) $\mathbf{11 + 4 - 3 < 11 + 2}$
The symbol < means **'is less than'**.
Left-hand side = 12 and **Right-hand side = 13**.

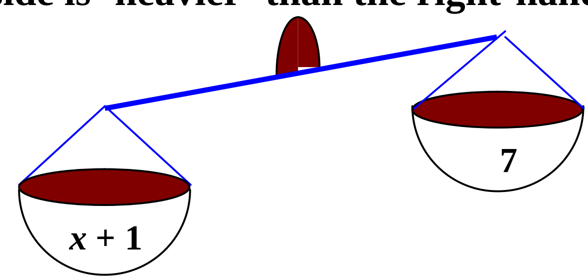
(ii) $\mathbf{2 \times (11 - 5) > 2 \times 7 - 4}$
The symbol > means **'is greater than'**.
Left-hand side = 12 and **Right-hand side = 10**.

Now, we shall look at some **algebraic inequations** and find the **possible values** of the unknown quantity in each.

Examples of Algebraic Inequations:

(i) $x + 1 > 7$

If we put these qualities on the scales, they will **not** balance as the **left-hand side is 'heavier' than the right-hand side.**



Subtract 1 from each side:

$$\begin{array}{rcr} x + 1 & > & 7 \\ -\ 1 & & -\ 1 \\ \hline x & > & 6 \end{array}$$

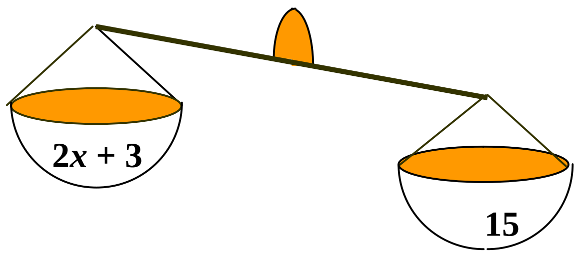
Now we have **solved the inequation**:

$$x + 1 > 7$$

and we have found the **solution** to be: $x > 6$.

(ii) Solve $\quad 2x + 3 < 15$

Again, these quantities would **not** balance on the scales, as the **left-hand side** is **'lighter'** than the **right-hand side** .



$$2x + 3 < 15$$

Subtract 3 from each side: $\quad \underline{-3} < \underline{-3}$

: $\quad 2x < 12$

Divide each side by 2: $\quad x < 6$

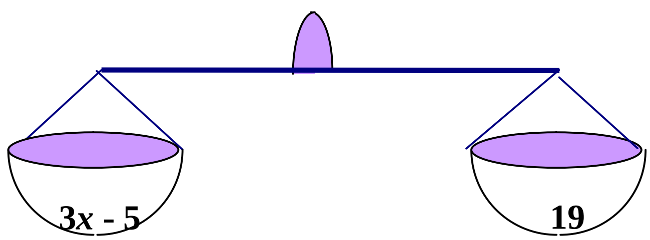
The **solution to the inequation**:

$$2x + 3 < 15$$

is $\quad x < 6$.

(iii) Solve $3x - 5 \leq 19$

(Note: The symbol ≤ means: ***"is less than or equal to".)***



$$3x - 5 \leq 19$$

Add 5 to each side: $\quad \underline{+5} \quad \underline{+5}$

$$3x \leq 24$$

Divide each side by 3: $\quad x \leq 8$

The **solution** to the inequation :

$$3x - 5 \leq 19$$

is $\quad x \leq 8$.

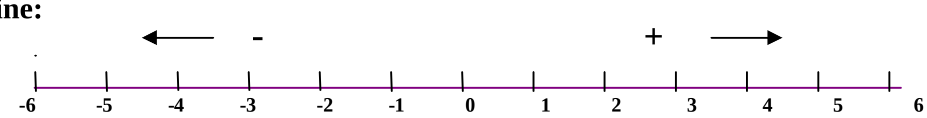
(i.e. x is less than or equal to 8).

THE NUMBER LINE

A **number line** may be used to **show solution sets** of **equations** and **inequations**. It is particularly helpful in the case of **inequations**, where ***x***, for example, can be **any** of a **range** of **values**.

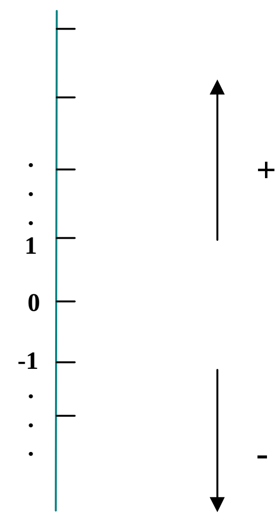
Number Line:

Horizontal Line:



OR

Vertical Line:



Example :

(a) **Solve $x + 2 < 5$,** (***x*** is a **whole** number)

and

(b) **Show** the **solution set** on a **number line**.

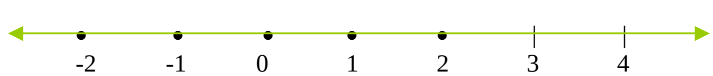
(a)

$$x + 2 < 5$$

Subtract 2 : $\underline{-2} \quad \underline{-2}$

$$x < 3$$

(b) **Number Line**:



EXERCISE 14

(Simple inequations and the number line.)

For each of the following **inequations**,
if x is a **whole number**,

(a) **solve** the **inequations**
and

(b) **show** the **solution** to each **inequation** in part (a) on a **number line** :

1. $x + 1 < 7$

2. $2x + 1 > 7$

3. $3x + 1$ £ 7

4. $x + 1 > 0$

5. $2x + 2$ £ 0

6. $x - 1 < 0$

7. $2x - 2$ ³ 0

8. $2x + 1 < 3x$

9. $2x - 1 > 3x$

10. $2x - 1$ £ $3x - 2$

11. $6x + 1$ ³ $4x + 5$

12. $3x - 2 < 6x - 8$

EXERCISE 14 - ANSWERS

(a)

1. $x + 1 < 7$
Subtract 1 : $x < 6$

2. $2x + 1 > 7$
Subtract 1 : $2x > 6$
Divide by 2 : $x > 3$

3. $3x + 1 \le 7$
Subtract 1 : $3x \le 6$
Divide by 3 : $x \le 2$

4. $x + 1 > 0$
Subtract 1 : $x > -1$

5. $2x + 2 \le 0$
Subtract 2 : $2x \le -2$
Divide by 2 : $x \le -1$

6. $x - 1 < 0$
Add 1 : $x < 1$

7. $2x - 2 \ge 0$
Add 2 : $2x \ge 2$
Divide by 2 : $x \ge 1$

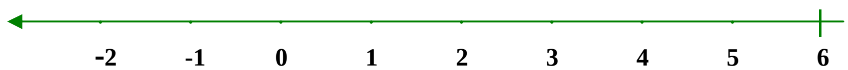
8. $2x + 1 < 3x$
Subtract 2x : $1 < x$
i.e. $x > 1$

9. $2x - 1 > 3x$
Subtract 2x : $-1 > x$
i.e. $x < -1$

10. $2x - 1 \le 3x - 2$
Add 2 : $2x + 1 \le 3x$
Subtract 2x : $1 \le x$
i.e. $x \ge 1$

11. $6x + 1 \ge 4x + 5$
Subtract 1 : $6x \ge 4x + 4$
Subtract 4x : $2x \ge 4$
Divide by 2 : $x \ge 2$

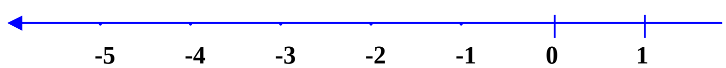
12. $3x - 2 < 6x - 8$
Add 8 : $3x + 6 < 6x$
Subtract 3x : $6 < 3x$
Divide by 3 : $2 < x$ (i.e. $x > 2$)

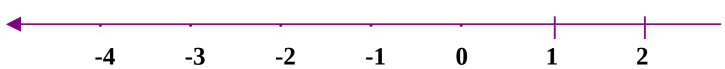
(b) 1.


2.

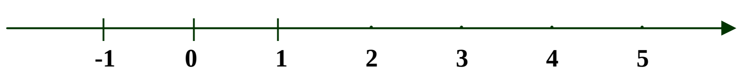

3.

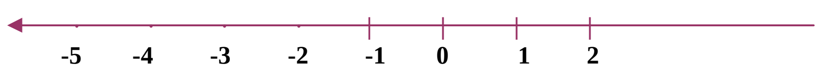

4.

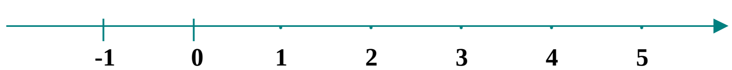

5.


6.


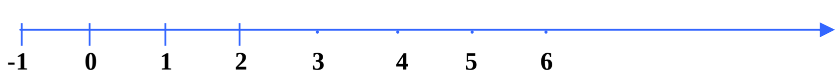
7.


8.


9.


10.


11.


12.


SECTION 15

SIMULTANEOUS EQUATIONS

SIMULTANEOUS EQUATIONS

If we have **two** equations, each containing **2 unknown quantities**,
it is possible to find a **solution** that satisfies **both equations simultaneously**, which means '**at the same time'**.

Equations of this type are called **'Simultaneous Equations'**.

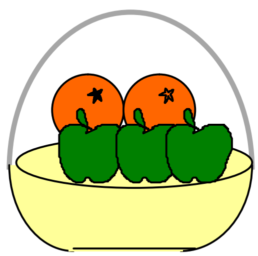
Eg. **(i)** If **2 oranges** and **3 apples** together cost **95p** and **1 orange** and **3 apples** together cost **70p**, find the cost of:

(a) 1 orange and

(b) 1 apple.

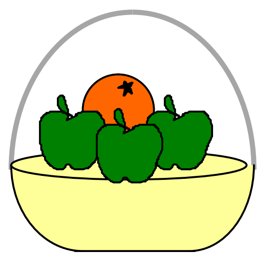
If we put the **two lots** of fruits into **separate baskets**, we have :

Cost = 95p



Basket 1

Cost = 70p



Basket 2

Notice that there are **3 apples** in each basket. If we take the **difference in the contents** of the baskets, it must be **equal** to the **difference** in the **costs** of the contents.

This gives

1 orange = 95p - 70p

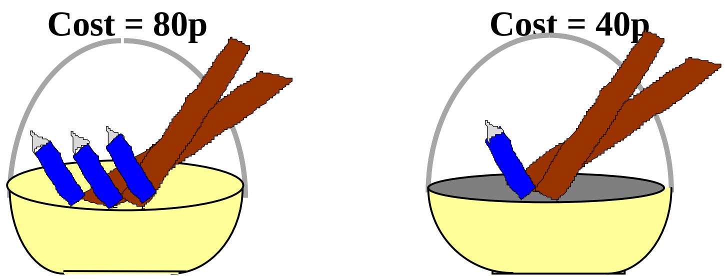
i.e. 1 orange costs **25p**.

Now, from Basket **2**, **1 orange** plus **3 apples** cost **70p**.
Therefore, **3 apples** must cost **70p - 25p**,
i.e. 3 apples cost **45p**.
Therefore, **1 apple** costs **15p**.

ANSWER: **(a)** **1 orange** costs **25p**; **(b)** **1 apple** costs **15p**.

Eg.(ii) If **3 pencils** and **2 rulers** together cost **80p** and **1 pencil** and **2 rulers** together cost **40p**, find the cost of:
(a) 1 pencil and
(c) 1 ruler.

Again, putting them into baskets, we have:



Basket 1 Basket 2

This time, we have **2 rulers** in each basket.

The **difference** in the **contents** of the baskets is **2 pencils** and the **difference in the costs** is **80p** - **40p** = **40p**.

Therefore **2 pencils** cost **40p**.
So **1 pencil** costs **20p**.

From Basket **2**:

1 pencil plus 2 rulers costs **40p**,

therefore, **2 rulers** must cost **40p - 20p = 20p**.

This gives: **1 ruler** costs **10p**.

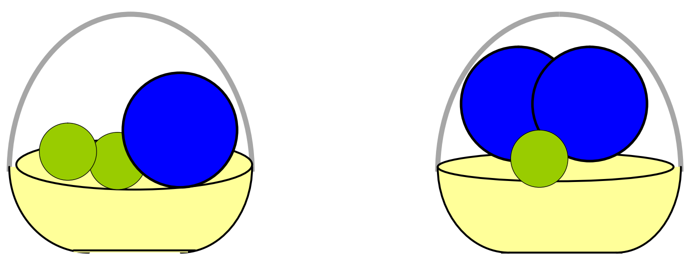
ANSWER: **(a)** **1 pencil** costs **20p**;
(b) **1 ruler** costs **10p**.

Eg.(iii) If **1 bat** and **2 balls** together cost **£2.00** and
2 bats and **1 ball** together cost **£2.50**,
find the cost of:

(a) 1 bat and

(b) 1 ball.

Cost = £2.00 **Cost = £2.50**



Basket 1 Basket 2

We need to have the **same number of bats** or the **same number of balls** in **each basket** and we do not have that situation here. What can we do?

We **do not** know the individual prices of the bats and balls but we
do know that we would be allowed to, say, **double** the **contents** of a basket, provided that we **also double** the **cost** of its contents.
In this case, this is what we **must** do before we can solve the problem.

We could make the **same number** of **bats** in each basket, if we **double** the contents of **Basket 1**.

Alternatively, we could make the **same number** of **balls** in each if we **double** the contents **of Basket 2**.

Eg.(iii) (Contd.)

Doubling Basket 2 gives:

4 bats and **2 balls** cost **£5.00**.

We shall call this **Basket 3**.

Now, the **difference** in the **contents** between **Basket 3** and **Basket 1** is **3 bats** and the **difference** in the **costs** is **£5 - £2** = **£3**.

Therefore, **1 bat** costs **£1.00** and **1 ball** costs **50p**.

ANSWER: **(a)1 bat** costs **£1.00** and **(b) 1 ball** costs **50p**.

SOLUTION OF SIMULTANEOUS EQUATIONS – ELIMINATION METHOD

The method used to solve the simultaneous equations in the **three examples** above is really a process of **elimination**. Notice that we were able to **eliminate** (i.e. "get rid of ") something from the baskets by **subtraction** of the **contents**, when we had **two baskets** containing the **same number** of it.

Whilst displaying the information in baskets helps greatly in grasping an understanding of this topic, the task is just as easy to perform (and much faster!), if we use proper **equations**, **instead** of baskets, to contain the information.

The idea is, of course, just the same. We can pretend that the equations are baskets if we want to visualise the problem!

We shall reconsider **Egs. (i)** to **(iii)** above, using the **algebraic** method of **elimination**, instead of the baskets, to **solve** the equations.

Eg. (i):

If we let x represent the cost, in pence, of an **orange** and y represent the cost in pence of an **apple**, we have:

$$2x + 3y = 95 \quad \ldots \quad \textbf{Equation 1}$$
$$1x + 3y = 70 \quad \ldots \quad \textbf{Equation 2}$$

Equation 1 – Equation 2 gives:

$$2x - x + 3y - 3y = 95 - 70$$

Simplifying this, we have: $x = 25$.

If we put $x = 25$ into **Equation 2**,
we have: $25 + 3y = 70$.

Now we must solve: $25 + 3y = 70$.

Subtract 25: $\underline{-25}$ $\underline{-25}$

$$3y = 45$$

Divide by 3: $y = 15$.

We have now **solved** the **simultaneous equations**:

$$2x + 3y = 95$$
and $x + 3y = 70$,

finding the **solution** to be:

$x = 25$
and $y = 15$.

This means, then, that:

an **orange** costs **25p**
and an **apple** costs **15p**,
as before.

Eg.(ii):

Letting p represent the cost, in pence, of a **pencil** and r represent the cost, in pence, of a **ruler**, we have:

$$\mathbf{3p + 2r = 80} \quad \ldots \quad \textbf{Equation 1}$$
$$\mathbf{1p + 2r = 40} \quad \ldots \quad \textbf{Equation 2}$$

Equation 1 – Equation 2 gives:

$$3p - p + 2r - 2r = 80 - 40$$

Simplifying this, we have: $2p = 40$

Divide by 2: $p = 20$.

If we put $p = 20$ into **Equation 2**, we have: $20 + 2r = 40$.

Now we must solve: $20 + 2r = 40$.

Subtract 20: $\underline{-20}$ $\underline{-20}$

$$2r = 20$$

Divide by 2: $r = 10$.

We have now **solved** the **simultaneous equations**:

$$3p + 2r = 80$$

and $p + 2r = 40$,

finding the **solution** to be:

$p = 20$

and $r = 10$.

This means that:

a **pencil** costs **20p**

and a **ruler** costs **10p**,

as before.

Eg.(iii):

Letting $\boldsymbol{x}$ represent the cost, in **pence**, of a **bat** and $\boldsymbol{y}$ represent the cost, in **pence**, of a **ball**, we have:

$$\mathbf{1}\boldsymbol{x} + \mathbf{2}\boldsymbol{y} = \mathbf{200} \quad \dots \quad \textbf{Equation 1}$$
$$\mathbf{2}\boldsymbol{x} + \mathbf{1}\boldsymbol{y} = \mathbf{250} \quad \dots \quad \textbf{Equation 2}$$

We are free to eliminate either $\boldsymbol{x}$ or $\boldsymbol{y}$, but in order to do so, we must first have the **same number** of the $\boldsymbol{x}$ or $\boldsymbol{y}$ in **both equations**. Clearly, we would have $\mathbf{2}\boldsymbol{x}$ in both equations, if we multiplied **Equation 1** by **2**. Alternatively, we could multiply **Equation 2** by **2** to give $\mathbf{2}\boldsymbol{y}$ in both equations. The amount of work is equal in either case.

Equation 1 ´ 2:

$$\mathbf{2}\boldsymbol{x} + \mathbf{4}\boldsymbol{y} = \mathbf{400} \quad \dots \quad \textbf{Equation 3}$$
$$\mathbf{2}\boldsymbol{x} + \mathbf{1}\boldsymbol{y} = \mathbf{250} \quad \dots \quad \textbf{Equation 2}$$

Equation 3 – Equation 2:

$$\mathbf{2}\boldsymbol{x} - \mathbf{2}\boldsymbol{x} + \mathbf{4}\boldsymbol{y} - \boldsymbol{y} = \mathbf{400} - \mathbf{250}$$

Simplifying gives: $\mathbf{3}\boldsymbol{y} = \mathbf{150}$

Divide by 3: $\boldsymbol{y} = \mathbf{50}$.

Now substitute y = 50 in Equation 2:

$$\begin{aligned} \mathbf{2}\boldsymbol{x} + \mathbf{50} &= \mathbf{250} \\ \underline{\quad -\mathbf{50}} &= \underline{-\mathbf{50}} \\ \mathbf{2}\boldsymbol{x} \qquad &= \mathbf{200} \\ \boldsymbol{x} \qquad &= \mathbf{100}. \end{aligned}$$

Subtract 50: (second line above)

Divide by 2: (last line above)

We have now **solved** the **simultaneous equations**:

$$\mathbf{2}\boldsymbol{x} + \mathbf{4}\boldsymbol{y} = \mathbf{400}$$

and $\mathbf{2}\boldsymbol{x} + \boldsymbol{y} = \mathbf{250}$,

finding the **solution** to be:

$\boldsymbol{x} = \mathbf{100p}$ or **£1.00**

and $\boldsymbol{y} = \mathbf{50p}$.

This means that a **bat** costs **£1.00** and a **ball** costs **50p**, as before.

Simultaneous Equations – Positive and Negative Values of x and y

The three practical examples of simultaneous equations considered earlier contained only **positive (**i.e. **+) values** of x and y, obviously because it would **not** make sense to have **negative (**i.e. **-) values**; we would **not** have **negative numbers** of **objects**, **£**, **people**, **animals**, etc.

We shall now look at **solution** of **simultaneous equations**, involving both **positive** and **negative** values of x and y.

Eg.(i) **Solve** the following set of equations:

$$x + y = 3 \quad \ldots \quad \textbf{Equation 1}$$
$$x - y = 1 \quad \ldots \quad \textbf{Equation 2}$$

If we wish to eliminate y, we must **add** these equations, since $y + (-y) = 0$. We could, of course, eliminate x by **subtracting** one equation from the other, but adding is usually easier than subtraction.

In this situation, I would recommend **adding** the equations to eliminate y, but I shall do it both ways to demonstrate that the **same answer** is obtained either way.

$$x + y = 3 \quad \ldots \quad \textbf{Equation 1}$$
$$x - y = 1 \quad \ldots \quad \textbf{Equation 2}$$

To eliminate y:

Equation 1 + Equation 2: $2x = 4$

***Divide by* 2** $x = 2$

Substitute in Equation 1: $2 + y = 3$

Subtract 2: $\underline{-2} \qquad \underline{-2}$

$y = 1$

The **solution** is, therefore:

$x = 2$,

and $y = 1$.

We shall now solve the equations in **Eg.(i)** by eliminating ***x***, instead of ***y***. **Remember** that, since there is **+1** of ***x*** in **each** equation, we must **subtract** the equations this time.

The working is as follows:

Solve the following set of equations:

$$\begin{array}{lrcl} & x + y = & 3 & \ldots \quad \textbf{Equation 1} \\ & x - y = & 1 & \ldots \quad \textbf{Equation 2} \\ \textit{Equation 1 – Equation 2:} & 2y = & 4 & (\text{N.B.} \quad +1y - (-1\,y) = +2y). \\ \textit{Divide by 2:} & y = & 2 & \\ & & & \\ \textit{Substitute in Equation 1:} & x + 2 = & 3 & \\ \textit{Subtract 2:} & \underline{-\,2} & \underline{-2} & \\ & x \quad = & 1 & \\ & & & \\ \textit{Substitute in Equation 1:} & 1 + y = & 3 & \\ \textit{Subtract 1:} & \underline{-1} \qquad & \underline{-1} & \\ & y = & 2 & \end{array}$$

Again we have the **solution**:

$$\begin{array}{lccc} & x & = & 1 \\ \text{and} & y & = & 2. \end{array}$$

Clearly, then, in simultaneous equations, you always have the **choice** of **eliminating** ***x*** or ***y***. However, the amount of **work** involved is **not** always **equal**, and mathematicians would normally opt for the quicker, and usually safer, option.

N.B. <u>**Rules for Eliminating**</u>

Signs the **same** (i.e. **both +** or **both -**) $\Rightarrow$ **subtract** the equations.

Signs **different** (i.e. **one +** and **one -**) $\Rightarrow$ **add** the equations.

Eg.(ii) **Solve** the following set of equations:

$$\begin{aligned} x + 2y &= 5 \quad \ldots \quad \textbf{Equation 1} \\ 2x - y &= 0 \quad \ldots \quad \textbf{Equation 2}. \end{aligned}$$

In this question, the amount of work is equal in eliminating ***x*** or ***y***.

We could eliminate ***x*** by **multiplying Equation 1** by **2**,
and then **subtracting** the equations.

Alternatively, we could eliminate ***y*** by **multiplying Equation 2** by **2**,
and then **adding** the equations.

Since adding is easier than subtraction, I opt for the
elimination of ***y***.

$$\begin{aligned} x + 2y &= 5 \quad \ldots \quad \textbf{Equation 1} \\ 2x - y &= 0 \quad \ldots \quad \textbf{Equation 2} \end{aligned}$$

Equation 2 ´ 2. $\quad 4x - 2y = 0 \quad \ldots \quad$ **Equation 3**:

Equation 1 + Equation 3: $\quad 5x = 5$

Divide by 5: $\quad x = 1$

Substitute in Equation 1: $\quad 1 + 2y = 5$

Subtract 1: $\quad \underline{-1} \qquad \underline{-1}$

$$2y = 4$$

Divide by 2: $\quad y = 2$

Therefore the **solution** is:

$$x = 1$$

and $\quad y = 2$.

Eg.(iii) **Solve** the set of equations:

$2x + 3y$	$=$	1	…	**Equation 1**
$3x - 2y$	$=$	8	…	**Equation 2**

To eliminate y:

Equation 1 ´ 2:	$4x + 6y$	$=$	2	…	**Equation 3**
Equation 2 ´ 3:	$9x - 6y$	$=$	24	…	**Equation 4**

Equation 3 + Equation 4:	$13x$	$=$	26
Divide by 13:	x	$=$	2
Substitute in Equation 1:	$4 + 3y$	$=$	1
Subtract 4:	$\underline{-4}$		$\underline{-4}$
	$3y$	$=$	-3
Divide by 3:	y	$=$	-1.

Therefore, the **solution** is:

$$x = 2$$

and $$y = -1.$$

Solution of Simultaneous Equations – Graphical Method

Simultaneous Equations may be solved **graphically** by using intersecting graphs**.**

If **two functions** are plotted on the **same axes,** using the **same scales,** the functions are **equal** to each other at the **point(**s) of intersection between their graphs**.**

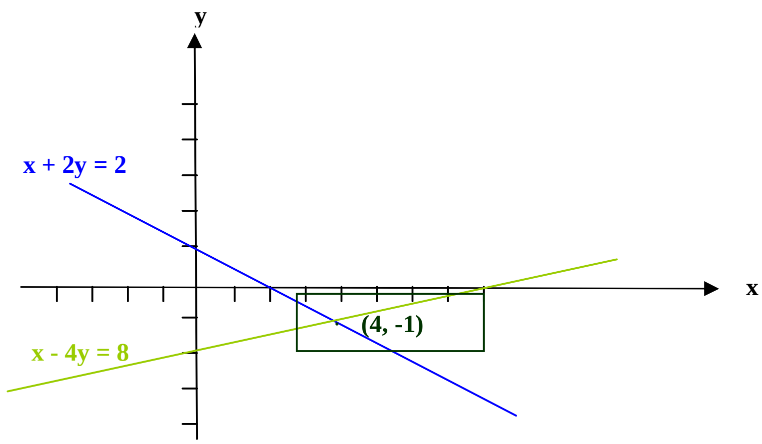
The examples overleaf demonstrate the method.

Use **graphical methods** to solve the following simultaneous equations:

(a) $x + 2y = 2$
$x - 4y = 8.$

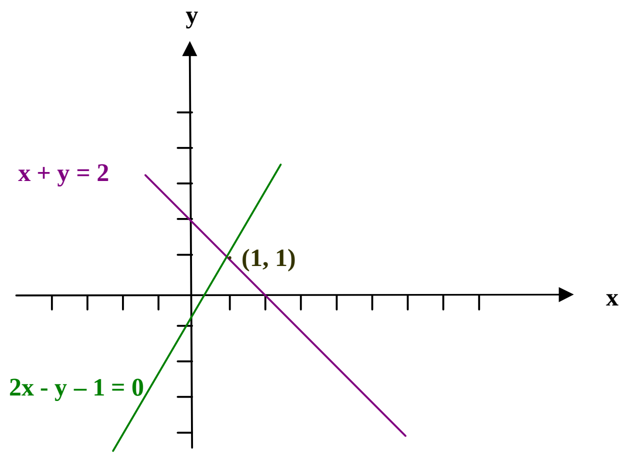
(b) $x + y - 2 = 0$
$2x - y - 1 = 0.$

Graph (i) (a)



The point of intersection is **(4, -1)**, giving the solution.

Graph (i) (b)



The point of intersection is **(1, 1)**, giving the solution.

EXERCISE 15

(Simultaneous Equations – solution by elimination.)

1. If **one** can of drink with **two** portions of chips cost **£1.10** and **three** cans of drink with **two** portions of chips cost **£1.90**, find the cost of:
   (a) **One** can of drink.
   (b) **One** portion of chips.

2. If **four** pens with **three** erasers cost **£1.30** and **three** pens with **four** erasers cost **£1.15**, find the cost of:
   (a) **One** pen.
   (b) **One** eraser.

3. If the **sum** of two numbers is **27** and the **difference** between them is **17**, find the numbers. (**Hint:** Let ***x*** be one number and ***y*** the other.)

4. If **five** cinema tickets with **four** theatre tickets cost **£72.50**, and **three** cinema tickets with **two** theatre tickets cost **£38.50**,

   find the cost of:
   (a) **One** cinema ticket.
   (b) **One** theatre ticket.

5. **Solve** the following simultaneous equations:

   (a)
   $$\begin{aligned} x + y &= 11 \\ x - y &= 7 \end{aligned}$$

   (b)
   $$\begin{aligned} 2x + 3y &= -4 \\ x - 3y &= 7 \end{aligned}$$

   (c)
   $$\begin{aligned} 4x - y &= 15 \\ 2x + 3y &= 11 \end{aligned}$$

5. (*Contd*.)

(d)
$$\begin{aligned} \mathbf{5}x - \mathbf{3}y &= \mathbf{13} \\ \mathbf{3}x + \mathbf{4}y &= \mathbf{2} \end{aligned}$$

(e)
$$\begin{aligned} \mathbf{7}x - \mathbf{2}y &= \mathbf{25} \\ \mathbf{2}x - y &= \mathbf{8} \end{aligned}$$

(f)
$$\begin{aligned} \mathbf{5}x - \mathbf{2}y &= \mathbf{7} \\ \mathbf{3}x + \mathbf{4}y &= \mathbf{-1} \end{aligned}$$

(g)
$$\begin{aligned} \mathbf{5}x - \mathbf{7}y &= \mathbf{-19} \\ \mathbf{7}x - \mathbf{5}y &= \mathbf{-17} \end{aligned}$$

(h)
$$\begin{aligned} x - \mathbf{6}y &= \mathbf{5} \\ \mathbf{3}x + \mathbf{2}y &= \mathbf{-5} \end{aligned}$$

(i)
$$\begin{aligned} -x + \mathbf{4}y &= \mathbf{-2} \\ \mathbf{5}x + y &= \mathbf{10} \end{aligned}$$

(j)
$$\begin{aligned} \mathbf{4}x + \mathbf{5}y &= \mathbf{-5} \\ \mathbf{3}x - \mathbf{10}y &= \mathbf{10} \end{aligned}$$

(k)
$$\begin{aligned} \mathbf{2}x - y &= \mathbf{0} \\ x + \mathbf{4}y &= \mathbf{9} \end{aligned}$$

6. **Solve** the simultaneous equations:

$$2x - y = -1$$

and $$3x + y = -4$$

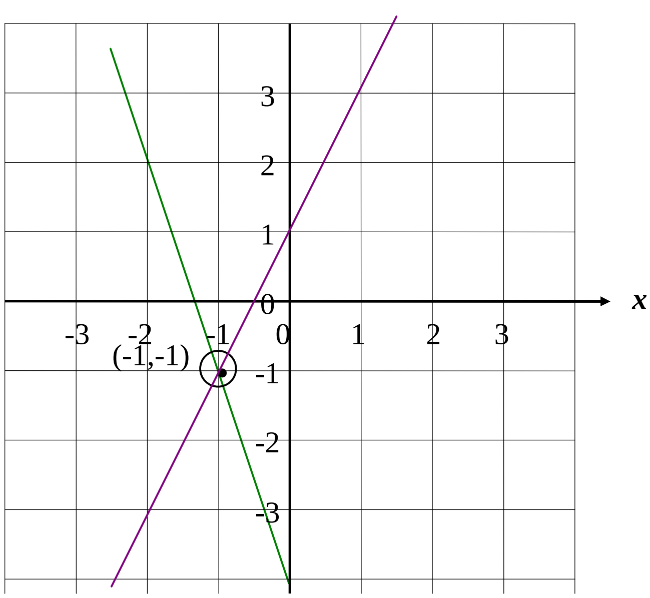
using a **graph**.

(Now solve graphically the simultaneous equations (a) to (k) in Question 5 – the answers should be the same as those found using the elimination method).

EXERCISE 15 - ANSWERS

1.	(a)	**40p**	(b)	**35p**
2.	(a)	**25p**	(b)	**10p**
3.	(a)	**22**	and	**5**
4.	(a)	**£4.50**	(b)	**£12.50**
5.	(a)	$x = 9$		$y = 2$
	(b)	$x = 1$		$y = -2$
	(c)	$x = 4$		$y = 1$
	(d)	$x = 2$		$y = -1$
	(e)	$x = 3$		$y = -2$
	(f)	$x = 1$		$y = -1$
	(g)	$x = -1$		$y = 2$
	(h)	$x = -1$		$y = -1$
	(i)	$x = 2$		$y = 0$
	(j)	$x = 0$		$y = -1$
	(k)	$x = 1$		$y = 2$

6.



(-1, -1) $\Rightarrow$ $x = -1, y = -1$

SECTION 16

STATISTICS

(Averages, Pie Chart, Bar Graph)

1. AVERAGES

There are **three types** of **average**, namely:

(i) Mean

This is the **normal arithmetic average**. Simply add up all the scores and divide by the number of them, to find the **mean** score.

(ii) Median

Before the median can be found, the **scores must be arranged in order of size**. In an **odd** number of scores, the **median** score is the **middle** one. In an **even** number of scores, the **median** is the **average of the two middle** scores.

(iii) Mode

The **modal** score is the score that **occurs the most often**. The mode is easily found, just by inspection.

WORKED EXAMPLE ON AVERAGES

In a survey, **20** children were asked how many magazines they read in each week.

The results of the survey were as follows:

2	6	0	4	1	3	4	2	0	4
2	4	1	1	7	3	4	5	6	2

2. TALLY CHART

Before they are **organised**, the results are called **raw data**. We organise raw data by listing the different scores in **ascending order of size** and then drawing up a **TALLY CHART**, as follows :

Score	Tally	Frequency
0	II	2
1	III	3
2	IIII	4
3	II	2
4	~~IIII~~	5
5	I	1
6	II	2
7	I	1

N.B. IIII = 4
and ~~IIII~~ = 5

(i) **Mean**

The **mean** number of magazines read per week

$$= \frac{\textbf{Total Number of Magazines Read}}{\textbf{No. of Children}}$$

$$= \frac{61}{20}$$

$$= \frac{6.1}{2}$$

$$= 3.05$$

= Therefore, **the mean number of magazines read per week is 3.05.**

(ii) **Median**

When the **20 scores** are arranged **in ascending order of size**, we have:

0 0 1 1 1 2 2 2 2 3 3 4 4 4 4 4 5 6 6 7

The **2 middle scores** in the 20 scores are, therefore, the **10th score** and the **11th score**.
= **3** and **3**

Average of 3 and 3 is

$$\frac{\mathbf{3 + 3}}{\mathbf{2}}$$

$$= \quad \mathbf{3}$$

Therefore, the **median number of magazines read per week is 3**.

(iii) Mode

The score that occurs the **most often** is **4**.
Therefore, **4** is the **modal number** of magazines read per week.

NOTE : If you are asked to find '**the average'** of a set of numbers, this is the **mean**. The median and mode are **not** given, unless they are **specifically** asked for.

3. PIE CHART

The **results of the survey on magazines** could be illustrated on a **PIE CHART**. The **pie chart is a circle divided into a number of sectors**, each of which displays **a proportion of the whole sample**.

The **size of the angle in each sector must be determined** before a pie chart can be drawn. Remember that there are **360° in the whole circle**.

In our survey, we have **20 children altogether**.

Therefore, **20 children** $=$ **360°**

and **1 child** $= \frac{\mathbf{360°}}{\mathbf{20}}$

i.e. **1 child** $=$ **18°**.

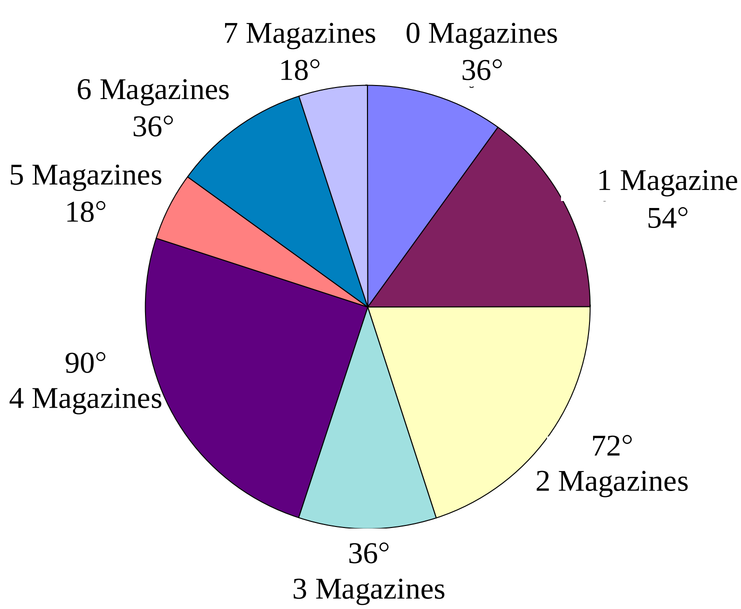
Next, we **change the numbers of children to degrees**, as follows:

No. of Magazines	No. of Children	Degrees
0	2	2 ´ 18° = 36°
1	3	3 ´ 18° = 54°
2	4	4 ´ 18° = 72°
3	2	2 ´ 18° = 36°
4	5	5 ´ 18° = 90°
5	1	1 ´ 18° = 18°
6	2	2 ´ 18° = 36°
7	1	1 ´ 18° = 18°
TOTALS	**20**	**360°**

Now we are ready to do the pie chart.

We use a **protractor to measure the angle** which we need to draw in **each sector**.

PIE CHART REPRESENTING RESULTS OF SURVEY ON MAGAZINES



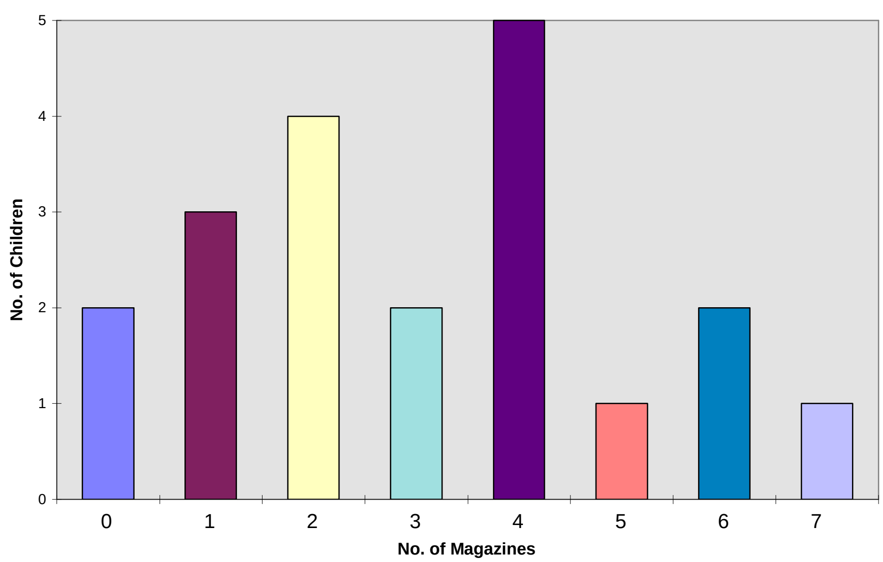
4. BAR GRAPH

The results of the survey on magazines could also be shown on a **BAR GRAPH**. In these graphs, the **data are represented by a series of bars, all of the same width**. The bars may be drawn **horizontally** or **vertically**. The **length** or **height** of **each bar** represents the **size of the figures**.

The data that we need to portray are as follows:

	No. of Magazines	No. of Children
Bar 1	0	2
Bar 2	1	3
Bar 3	2	4
Bar 4	3	2
Bar 5	4	5
Bar 6	5	1
Bar 7	6	2
Bar 8	7	1

Bar Graph representing Results of SURVEY ON MAGAZINES



5. RANGE

The **range** of a set of numbers is the difference between the **highest** and **lowest values** in the set.

Eg. Six children, playing a game, score:
5, 10, 4, 9, 4 and 7 points respectively.
Find the **range** of their scores.

Since the **lowest value** is **4** and the **highest value** is **10**, the **range** is **10 - 4 = 6** points.

EXERCISE 16

(Mean, Median, Mode, Range, Pie Chart, Bar Graph)

1. Find the **(a) mean**, **(b) median**, **(c) mode** and **(d) range** of each of the following sets of scores:

(i) 2, 0, 6, 3, 7, 3, 1
(ii) 9, 1, 0, 4, 7, 9
(iii) 2, 5, 3, 11, 5
(iv) 1, 7, 2, 0, 6, 2
(v) 8, 0, 3, 2, 4, 3, 5
(vi) 12, 8, 7, 8, 9, 6
(vii) 7, 9, 8, 0, 3, 5, 4, 7, 2
(viii) 13, 1, 6, 0, 6
(ix) 2, 10, 3, 0, 8, 5, 10
(x) 3, 2, 1, 0, 6, 2, 4

2.(a) Complete a **frequency table** for the following data, by counting the tally marks for each score :

Score	Tally	Frequency	
0	IIII	_________	(i)
2	I	_________	(ii)
5	~~IIII~~	_________	(iii)
9	~~IIII~~ I	_________	(iv)
12	~~IIII~~ III	_________	(v)

(b) Find the **(i) mean**, **(ii) median**, **(iii) mode** and **(iv) range** of the set of data in part **(a)**.

(c) Draw a **pie chart** to represent the data in part **(a)**.

(d) Draw a **bar graph** to represent the data in part **(a)**.

3. In a survey, **60 children** were asked to state their favourite colour.

15 children chose **red**,
25 children chose **blue**,
10 children chose **yellow**
and **10** children chose **green**.

(a) Draw a **pie chart** to represent this information.
(Remember : change the numbers of children into numbers of degrees first).

(b) Draw a **bar graph** to represent this information.
(Use a bar for each colour).

EXERCISE 16 – ANSWERS

1.	**(i)(a)** $3\frac{1}{7}$	**(b)** 3	**(c)** 3	**(d) 7**	**(ii)(a)** 5	**(b)** $5\frac{1}{2}$	**(c)** 9	**(d)** 9
	(iii)(a) $5\frac{1}{5}$	**(b)** 5	**(c)** 5	**(d)** 9	**(iv)(a)** 3	**(b)** 2	**(c)** 2	**(d)** 7
	(v)(a) $3\frac{4}{7}$	**(b)** 3	**(c)** 3	**(d)** 8	**(vi)(a)** $8\frac{1}{3}$	**(b)** 8	**(c)** 8	**(d)** 6
	(vii)(a) 5	**(b)** 5	**(c)** 7	**(d)** 9	**(viii)(a)** $5\frac{1}{5}$	**(b)** 6	**(c)** 6	**(d)** 13
	(ix)(a) $5\frac{3}{7}$	**(b)** 5	**(c)** 10	**(d)** 10	**(x)(a)** $2\frac{4}{7}$	**(b)** 2	**(c)** 2	**(d)** 6

2. **(a)(i)** 4 **(ii)** 1 **(iii)** 5 **(iv)** 6 **(v)** 8 **(b)(i)** $7\frac{3}{8}$ **(ii)** 9 **(iii)** 12

   (c) Angles in sectors : 60°, 15°, 75°, 90°, 120° respectively.

   (d) Heights of Bars : 4 1 5 6 8 respectively.

3. **(a)** Angles in Sectors : 90°, 150°, 60°, 60° respectively.

   (b) Heights of Bars : 15, 25, 10, 10 respectively.

SECTION 17

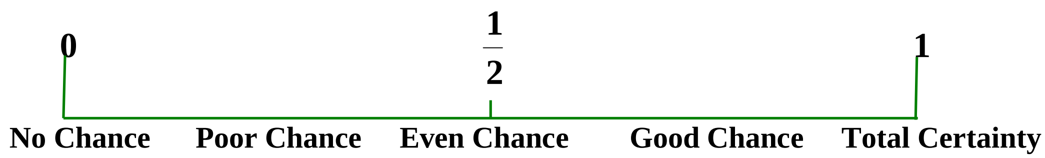
PROBABILITY

Probability is the **law of chance**. It assesses the likelihood of a particular event or occurrence.

The probability of a person living to be 120 years old is very small, perhaps $\frac{\mathbf{1}}{\mathbf{60,000,000}}$. The probability that a new baby will be a girl is $\frac{\mathbf{1}}{\mathbf{2}}$. The probability of a triangle having 4 sides is **0**. The probability of a hexagon having 6 sides is **1**.

No chance implies **Probability 0**.
Total certainty implies **Probability 1**.
Most probabilities lie somewhere between 0 and 1.

The **probability scale** looks like this:



Fractions may be used, for example $\frac{\mathbf{1}}{\mathbf{2}}$ **probability**.

Decimals may be used, for example **0.5 probability**.

Percentages may be used, for example **50% probability**.

N.B. **Ratio may not** be used, for example **1 : 2** is **WRONG**.

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WORKED EXAMPLES ON PROBABILITY

(i) A **fair coin** is tossed.

The probability of a **Head** is $\frac{1}{2}$**.** The probability of a **Tail** is $\frac{1}{2}$.

(ii) A **biased** (unfair) **coin** is tossed.

If the probability of a **Tail** is $\frac{3}{4}$, then the probability of a **Head** is $\frac{1}{4}$. (**The 2 probabilities must add up to 1**).

(iii) A **fair die** is rolled.

(a) The probability of a **1** is $\frac{1}{6}$.

(b) The probability of a **2** is $\frac{1}{6}$.

(c) The probability of a **3** is $\frac{1}{6}$.

(**d)** The probability of a **4** is $\frac{1}{6}$.

(e) The probability of a **5** is $\frac{1}{6}$.

(f) The probability of a **6** is $\frac{1}{6}$.

(g) The probability of an **even number** is $\frac{3}{6} = \frac{1}{2}$, since there are **even numbers on 3 of the 6 faces**.

(h) The probability of an **odd number** is also $\frac{3}{6} = \frac{1}{2}$.

(i) The probability of a **prime number** is $\frac{3}{6} = \frac{1}{2}$, since there are **prime numbers on 3 of the 6 faces** (namely 2, 3 and 5).

(j) The probability of a **multiple of 3** is $\frac{2}{6} = \frac{1}{3}$, since there are **multiples of 3 on 2 of the 6 faces** (namely 3 and 6).

(iv) A box contains **10 counters**, of which **4 are red, 3 are blue, 2 are yellow** and **1 is green**. **One counter** is chosen, at random, from the box.

(a) The probability that it is **red** is $\mathbf{\frac{4}{10}} = \mathbf{\frac{2}{5}}$.

(b) The probability that it is **blue** is $\mathbf{\frac{3}{10}}$.

(c) The probability that it is **yellow** is $\mathbf{\frac{2}{10}} = \mathbf{\frac{1}{5}}$.

(d) The probability that it is **green** is $\mathbf{\frac{1}{10}}$.

(e) The probability that it is **white** is **0**. (**No white counters** in the box !)

(f) The probability that it is **red or blue or yellow or green** is **1**. (This is **certain** !)

EXERCISE 17

(Probability)

1. Write out the **probability** of each of the following:

(i) A triangle having 5 sides.
(ii) A new baby being a girl.
(iii) A square having 4 sides.
(iv) A person being 200 years old.
(v) A dog having 4 legs.
(vi) A man having 3 legs.
(vii) A prime number being divisible by 10.
(viii) An elephant having 2 trunks.
(ix) A head when a fair coin is tossed.
(x) A 3 when a fair die is rolled.

2. A tray contains **20** bulbs:

10 narcissi, **6 daffodils** and **4 tulips**. **If 1 bulb is picked at random** from the tray, find the **probability** that it is:

(i) a narcissus
(ii) a daffodil
(iii) a tulip
(iv) a narcissus or a daffodil
(v) a daffodil or a tulip
(vi) a narcissus or a tulip
(vii) a narcissus, a daffodil or a tulip
(viii) a crocus.

3. A box contains **12** coloured counters:
6 are green, **2 are blue**, **3 are yellow** and **1** is **white**. If **1 counter is chosen at random from the box**, find the **probability** that it is:

(i) green
(ii) blue
(iii) yellow
(iv) white
(v) black
(vi) green or blue or yellow
(vii) green or blue or yellow or white.

EXERCISE 17 - ANSWERS

1. **(i)** 0 **(ii)** $\frac{1}{2}$ **(iii)** 1 **(iv)** 0 **(v)** 1 **(vi)** 0 **(vii)** 0 **(viii)** 0 **(ix)** $\frac{1}{2}$ **(x)** $\frac{1}{6}$

2. **(i)** $\frac{1}{2}$ **(ii)** $\frac{3}{10}$ **(iii)** $\frac{1}{5}$ **(iv)** $\frac{4}{5}$ **(v)** $\frac{1}{2}$

   (vi) $\frac{7}{10}$ **(vii)** 1 (certain) **(viii)** 0 (No crocus in tray)

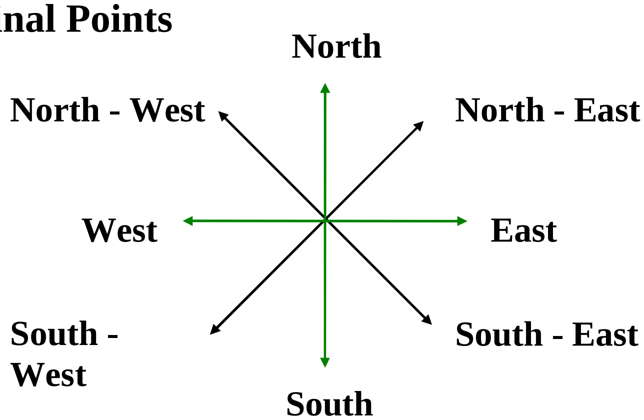
3. **(i)** $\frac{1}{2}$ **(ii)** $\frac{1}{6}$ **(iii)** $\frac{1}{4}$ **(iv)** $\frac{1}{12}$ **(v)** 0 (No black in box) **(vi)** $\frac{11}{12}$

   (vii) 1 (certain)

SECTION 18

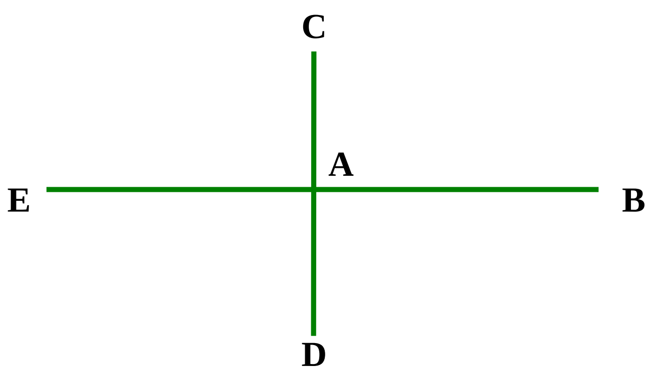
DIRECTIONS – CARDINAL POINTS & BEARINGS

1.Cardinal Points



WORKED EXAMPLES

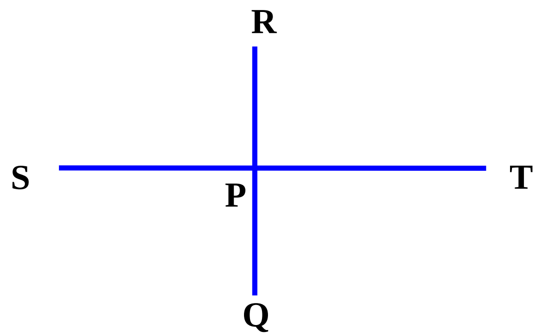
1. **A** is **due West of B** and **due South of C**.
   D is **due South of A**. **E** is **due West of A**.



.

(**a**) What direction is B from A?
(**b**) What direction is C from B?
(**c**) What direction is E from C?
(**d**) What direction is D from E?
(**e**) What direction is C from D?
(**f**) What direction is D from C?
(**g**)(**i**) Which point is furthest North?
(**ii**) Which point is furthest South?
(**iii**) Which point is furthest East?
(**iv**) Which point is furthest West?

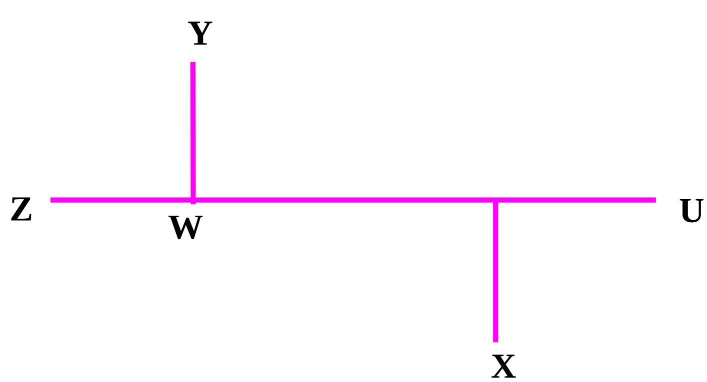
2. **5 points** are placed so that **P** is **due North of Q** and **due South of R**. **S** is **due West of P**, which is **due West of T**.



(**a**) What direction is T from S?
(**b**) What direction is P from R?
(**c**) What direction is R from Q?
(**d**) What direction is T from Q?
(**e**) What direction is S from R?
(**f**) Which point is furthest North?
(**g**) Which point is furthest South?
(**h**) Which point is furthest East?
(i) Which point is furthest West?

3. **U, V, W, X, Y and Z** are **six points** on a map. **V** is **due West of U** and **due North of X**. **W** is **due West of V and due East of Z**. It is also **due South of Y**.

(**a**) What is the most Northerly point?
(**b**) What is the most Southerly point?
(**c**) What is the most Easterly point?
(c) What is the Westerly point?



ANSWERS TO WORKED EXAMPLES

1.

(**a**)	Due - East		(**b**)	North - West.	(**c**)	South - West.	
(**d**)	South - East.		(**e**)	Due - North.	(f)	Due - South.	
(**g**)	(**i**) C	(**ii**)	D	(**iii**) B	(**iv**)	E	

2.

(**a**)	Due East.	(**b**)	Due South.	(**c**)	Due North.
(**d**)	North - East.	(**e**)	South - West.	(**f**)	R
(**g**)	Q	(**h**)	T	(**i**)	S

3.

(**a**) Y (**b**) X (**c**) U (**d**) Z

2. Three-figure Bearings

As an alternative to using the four cardinal directions, North, East, South and West, **three-figure bearings** may be used.

These are measured **from North** in **clockwise** direction.

(Remember: '**N** for **Nought'** and '**N** for **North'**).

Compare **Diagram 1** with **Diagram 2** (below).

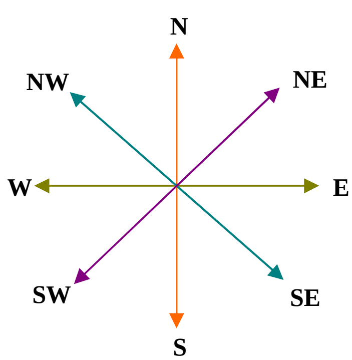


Diagram 1

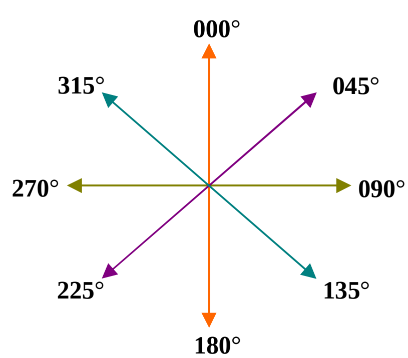


Diagram 2

Sample Question on Bearings

Two children, Angela and Matthew, set out from a **starting point P**. Angela walks for **30 minutes** on a bearing of **210° from P**, at an average speed of **4 km/hr**.
Matthew cycles for **30 minutes** on a bearing of **125° from P**, at an average speed of **8 km/hr**.

Using a **scale of 2 cm = 1 km**., draw a diagram to represent the journeys of Angela and Matthew and the answer to the following questions:
(a) How far has Angela travelled from P at the end of the 30 minutes ?
(b) How far has Matthew travelled from P at the end of the 30 minutes ?
(c) Use your ruler to measure the **distance between Angela and Matthew** at the end of the 30 minutes.
(d) The bearing of Angela from Matthew at the end of the 30 minutes.
(e) The bearing of **Matthew from Angela** at the end of the 30 minutes.
(N.B. Remember that the bearings are measured from the Northern line and going <u>clockwise</u> each time)

When you have completed the question, compare your answers with those below.

Answers to Sample Questions on Bearings

(a) 2 km.
(b) 4 km.
(c) $\frac{8.6cm}{2}$ = 4.3 km.
(d) 287°
(e) 107°

EXERCISE 18

1. **6 children, Alan, Bea, Cal, Dan, Ellie and Fran** are taking part in a canoeing navigation course.
Their starting positions are as follows :

Alan is **due South of Bea** and **due West of Cal**.
Cal is **due West of Dan**.
Ellie is **due West of Bea** and **due South of Fran**.

Draw a diagram showing their positions and then answer the following questions:

(i) What direction is Bea from Alan?
(ii) What direction is Cal from Alan?
(iii) What direction is Dan from Cal?
(iv) What direction is Bea from Ellie?
(v) What direction is Fran from Ellie?
(vi) Who is the furthest North?
(vii) Who is the furthest East?
(viii) If Alan wishes to paddle his canoe to Dan's position, in which direction will he have to paddle?
(ix) If Ellie wishes to paddle to Alan's position, in which direction will she have to paddle?
(x) If Cal wishes to paddle to Fran's position, in which direction will he have to paddle?
(xi) If Cal has to paddle to a position due North of Alan, in which direction will he have to paddle?
(xii) If Dan has to paddle to a position due South of Alan, in which direction will he have to paddle?
(xiii) If Bea has to paddle to Fran's position, in which direction will she have to paddle?

2. **V** is **due West of W**, **due East of X**, **due South of Y** and **due North of Z**.

Draw a diagram to represent the points V, W, X, Y and Z and then answer the following questions:

(i) What direction is Y from X?

(ii) What direction is Y from W?
(iii) What direction is Z from X?
(iv) What direction is Z from W?
(v) What direction is W from X?
(vi) What direction is Y from Z?

3. Two boats, A and B, sail from a port P.
A travels on a bearing of 150° and B travels on a bearing of 060° from P.
One hour after leaving P, A has sailed 35 km and B 40 km.
Using a ruler and protractor, draw an accurate diagram to represent the journeys of the two boats.
(Use a scale of 1cm = 5km)

From your diagram, answer the following questions:

(a) How far is Boat A from Boat B after one hour of sailing?
(b) What is the bearing of Boat A from Boat B after one hour?
(c) What is the bearing of Boat B from Boat A after one hour?
(d) What bearing would Boat A have to follow to return to Port P?
(e) What bearing would Boat B have to follow to return to Port P?

4. A point A is on a bearing of 180° from B and due West of C.

Copy the diagram below and mark the position of A.

5. **6** children**, Anne, Ben, Con, Dot, Ed and Flo** are taking part in orienteering.
Their starting positions are as follows:

Anne is **due North of Ben** and **due West of Con**.
Con is **due South of Dot**.
Ed is **due West of Ben** and **due East of Flo**.

Draw a diagram showing their positions and then answer the following questions:

(i) What direction is Ben from Anne?
(ii) What direction is Con from Anne?
(iii) What direction is Dot from Con?
(iv) What direction is Ben from Ed?
(v) What direction is Flo from Ed?
(vi) Who is the furthest North?
(vii) Who is the furthest West?
(viii) If Anne wishes to walk to Dot's position, in which direction will she have to walk?
(ix) If Ed wishes to walk to Anne's position, in which direction will he have to walk?
(x) If Dot wishes to walk to Ed's position, in which direction will she have to walk?
(xi) If Con has to walk to a position due North of Anne, in which direction will he have to walk?
(xii) If Dot has to walk to a position due South of Anne, in which direction will she have to walk?
(xiii) If Ben has to walk to Flo's position, in which direction will he have to walk?

6. Draw a diagram to represent the points A, B, C, D and E

and then answer the following questions:

A is **due West** of **B**, **due East** of **C**, **due South** of **D** and **due North** of **E**.

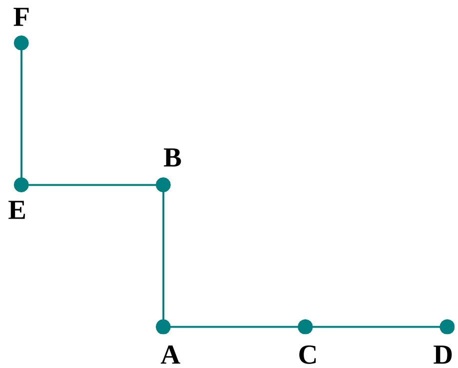
(i) What direction is D from C?
(ii) What direction is D from B?
(iii) What direction is E from C?
(iv) What direction is E from B?
(v) What direction is B from C?
(vi) What direction is D from E?

7. A point A is on a bearing of 160° from B and due West of C.

Draw a diagram and mark the position of A.

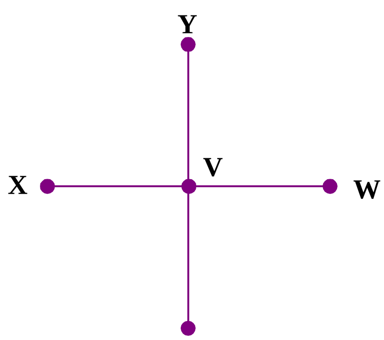
EXERCISE 18 - ANSWERS

1.



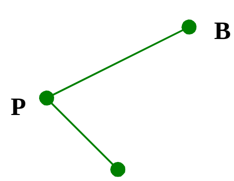
(i)	Due North	**(ii)**	Due East
(iii)	Due East	**(iv)**	Due East
(v)	Due North	**(vi)**	Fran
(vii)	Dan	**(viii)**	Due East
(ix)	South-East	**(x)**	North-West
(xi)	North-West	**(xii)**	South-West
(xiii)	North- West		

2.



(i)	North-East	**(ii)**	North-Wast
(iii)	South-East	**(iv)**	South-West
(v)	Due East	**(vi)**	Due North

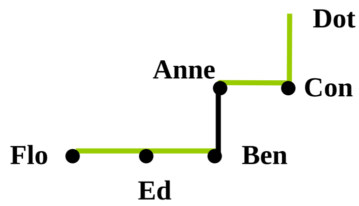
3. (a) 52.5km
(b) 200º
(c) 020º
(d) 330º
(e) 240º



N.B. not drawn to scale.

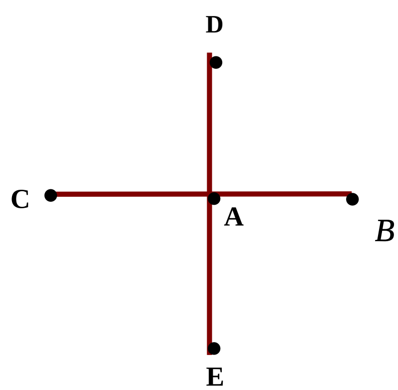
4.

5.



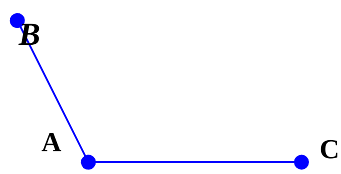
(i)	Due South	**(ii)**	Due East
(iii)	Due North	**(iv)**	Due East
(v)	Due West	**(vi)**	Dot
(vii)	Flo	**(viii)**	North-East
(ix)	North-East	**(x)**	South-West
(xi)	North-West	**(xii)**	South-West
(xiii)	Due West		

6.



(i)	North-East	**(ii)**	North-West
(iii)	South-East	**(iv)**	South-West
(v)	Due East.	**(vi)**	Due North.

7.



N.B. Not drawn accurately.

SECTION 19

WEIGHTS & MEASURES
- METRIC/IMPERIAL CONVERSIONS
TEMPERATURE SCALES

Prefixes used in metric weights and measures :

milli	=	$\mathbf{\frac{1}{1000}}$	(Latin : ***mille*** means **1000**)
centi	=	$\mathbf{\frac{1}{100}}$	(Latin : ***centum*** means **100**)
deci	=	$\mathbf{\frac{1}{10}}$	(Latin : ***decem*** means **10**)
kilo	=	1000 times	(Greek : ***kilo*** means **1000 times**)

Weight (Mass)

Conversions

(**≈** means **is approximately equal to**)

1000 mg	=	1 g	25 g	≈	1 ounce
1000 g	=	1 kg	1 kg	≈	2.2 pounds
1000 kg	=	1 tonne	1 tonne	≈	1 ton

(**mg** stands for **milligram**, **g** for **gram** and **kg** for **kilogram**.)

Capacity

1000 ml	=	1*l*	1l	≈	$1\frac{3}{4}$ pints
100 cl	=	1*l*	$4\frac{1}{2}$l	≈	1 gallon
10 ml	=	1 c*l*			

(**ml** stands for **millilitre**, **cl** for **centilitre** and **l** for **litre**.)

Length

1000mm	=	1m			
100cm	=	1m	30 cm	≈	1 foot
10mm	=	1cm	$2\frac{1}{2}$ cm	≈	1 inch
1000m	=	1km	1 m	≈	1 yard
			8 km	≈	5 miles

(**mm** stands for **millimetre**, **cm** for **centimetre**, **m** for **metre** and **km** for **kilometre.**)

Temperatures

2 scales:

(i) **Fahrenheit**

(**ii**) **Celsius** (or **Centigrade**)

Water	**°C**	**°F**
Freezing Point	0°	32°
Boiling Point	100°	212°

To change:

(**i**) °**Celsius to** °**Fahrenheit**:

°C $\times$ 9
$\div$ 5
$+$ 32

(**ii**) °**Fahrenheit to** °**Celsius**:

°F - 32
$\times$ 5
$\div$ 9

(i.e. **the above procedure in reverse**)

E.g. (i) To change **20°C** to **°F**, we have:

$$\begin{array}{r} 20 \\ \underline{\times\ 9} \\ 5\ \overline{)\ 180} \\ 36 \\ +\ \underline{32} \\ 68 \end{array}$$

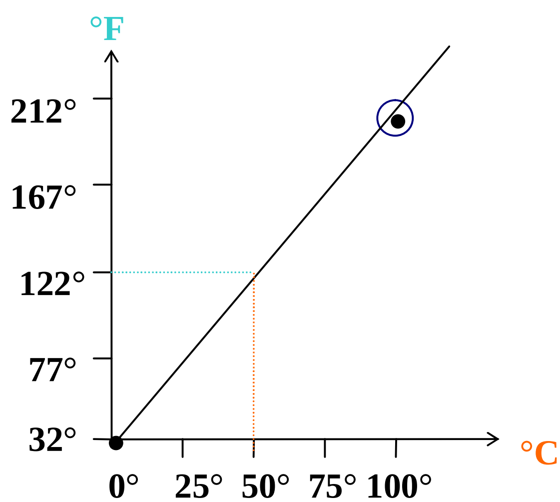
i.e. **20°C = 68°F**.

E.g. (ii) To change **68°F** to **°C**, we have:

$$\begin{array}{r} 68 \\ -\ \underline{32} \\ 36 \\ \underline{\times\ 5} \\ 9\ \overline{)\ 180} \\ 20 \end{array}$$

i.e. **20°C = 68°F**.

A **conversion graph** may be used when **changing one temperature to the other**. Plot (**0°C, 32°F**) and (**100°C, 212°F**) and join these to form a straight line.



Temperature Conversion Graph

E.g. Using Graph, **50° C = 122°F**.

EXERCISE 19

1. Change the following to m:

(i)	50 cm.	**(ii)**	3 cm.	**(iii)**	50 mm.
(iv)	3 mm.	**(iv)**	150 mm.	**(vi)**	150 cm.
(vii)	1500 mm.	**(viii)**	1500 cm.	**(ix)**	1.6 km
(x)	0.755 km.				

2. Change the following to mm:

(i)	5 cm.	**(ii)**	5 m.	**(iii)**	1.2 km.
(iv)	1.06 m.	**(v)**	0.005 km.	**(vi)**	0.5 m.
(vii)	1.2 cm.	**(viii)**	0.16 cm.	**(ix)**	0.001 km.
(x)	0.03 km.				

3. Change the following to ml:

(i)	5 c*l*.	**(ii)**	5 *l*.	**(iii)**	1.2 *l*.
(iv)	0.05 c*l*.	**(v)**	0.002 *l*.	**(vi)**	12.1 c*l*.
(vii)	12.1 *l*.	**(viii)**	0.08 *l*.	**(ix)**	130 c*l*.
(x)	130 *l*.				

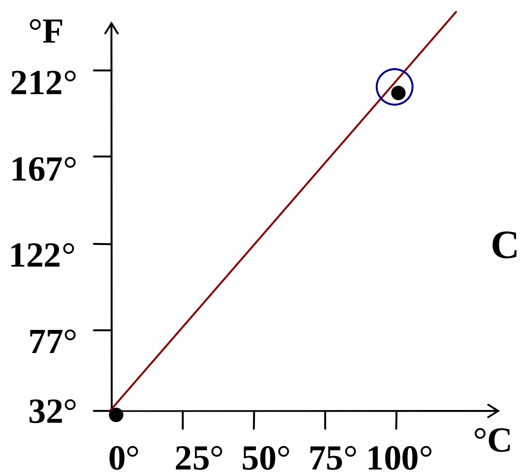
4. Change the following to *l*:

(i)	50 c*l*.	**(ii)**	5 m*l*.	**(iii)**	1.2 c*l*.
(iv)	0.05 m*l*.	**(v)**	0.004 c*l*	**(vi)**	400 c*l*.
(vii)	0.2 c*l*.	**(viii)**	0.2 m*l*	**(ix)**	12.1 c*l*.
(x)	12.1 m*l*.				

5.(**a**) Change the following temperatures:

(i)	10°C to °F.	**(ii)**	86°F to °C.
(iii)	40°C to °F.	**(iv)**	140°F to °C.
(v)	70°C to °F.	**(vi)**	194°F to °C.

(**b**) Use the conversion graph below to find the answers to part (**a**) - they should be the same or very close.
(Show your readings by drawing broken lines on the graph)



Temperature Conversion Graph

EXERCISE 19 - ANSWERS

1.	(**i**)	0.5(0) m.	(**ii**)	0.03 m.	(**iii**)	0.05(0) m.		
	(**iv**)	0.003 m.	(**v**)	0.15(0) m.	(**vi**)	1.5(0) m.		
	(**vii**)	1.5(00) m.	(**viii**)	15 m.	(**ix**)	1600 m.	(**x**)	755 m.
2.	(**i**)	50 mm.	(**ii**)	5000 mm.	(**iii**)	1200000 mm.		
	(**iv**)	1060 mm.	(**v**)	5000 mm.	(**vi**)	500 mm.		
	(**vii**)	12 mm.	(**viii**)	1.6 mm.	(**ix**)	1000 mm.	(**x**)	30000 mm.
3.	(**i**)	50 m*l*.	(**ii**)	5000 m*l*.	(**iii**)	1200 m*l*.		
	(**iv**)	0.5 m*l*.	(**v**)	2 m*l*.	(**vi**)	121 m*l*.		
	(**vii**)	12100 m*l*.	(**viii**)	80 m*l*.	(**ix**)	1300 m*l*.	(**x**)	130000 m*l*.
4.	(**i**)	0.5(0) *l*.	(**ii**)	0.005 *l*.	(**iii**)	0.012 *l*.		
	(**iv**)	0.00005 *l*.	(**v**)	0.00004 *l*.	(**vi**)	4 *l*.	(**vii**)	0.002 *l*.
	(**viii**)	0.0002 *l*.	(**ix**)	0.121 *l*.	(**x**)	0.0121 *l*.		
5.(**a**)	(**i**)	50°F	(**ii**)	30°C	(**iii**)	104°F	(**iv**)	60°C
	(**v**)	158°F	(**vi**)	90°C				

(**b**) Graph answers same as, or close to, those in part (**a**) above.

SECTION 20

SIMPLE & COMPOUND INTEREST
APPRECIATION/DEPRECIATION

Simple & Compound Interest

PRINCIPAL is the amount of money invested or borrowed.
INTEREST is the money 'earned' by the principal.
Interest is calculated at a **RATE %**, taking into account how many **years** the investment or loan lasts.

There are two types of interest, namely:

(i) **Simple Interest**
and
(ii) **Compound Interest**.

(i) Simple Interest

Interest is the profit earned from an investment. If money is borrowed, then the person who borrows it must pay interest to the lender. The money that is invested or lent is called the *principal*. The percentage return per annum is the *rate %*. The length of life (in years) of the loan or investment is called the *time*. The total of the interest added to the principal is called the *amount*. The amount is, therefore, the total sum remaining invested after a certain number of years. With simple interest, the *principal stays the same*, regardless of the length of time for the investment or loan.

E.g.1 £200 is invested for 3 years at 10% per annum simple interest. Find the amount at the end of the 3 years.

	Principal	=	**£200**			
	Rate	=	**10%**			
	Time	=	**3 years**			
	Interest	=	**10%** of **£200**	´	**3**	
		=	**£20** ´ **3**	=	**£60**.	
\	**Amount**	=	**£200 + £60**	=	**£260**.	

Simple Interest Formula

P = Principal
R = Rate %
Y = No. of Years

I = Interest

$$\boxed{I = \frac{PRY}{100}}$$

The Simple Interest Formula is convenient to use in calculations.
Applying the formula to the example above, we have:

P = £200

R = 10 %

Y = 3

$$I = \frac{200 ´ 10 ´ 3}{100} = \frac{6000}{100} = £60.$$

\ **Amount = £200 + £60 = £260** (as before.)

The Simple Interest Formula can be transposed to give:

$$\boxed{P = \frac{100I}{RY}}$$

$$\boxed{R = \frac{100I}{PY}}$$

$$\boxed{Y = \frac{100I}{PR}}$$

E.g. 2 Find the sum of money which needs to be invested at 10% per annum simple interest to earn £60 interest at the end of 3 years.

$$\mathbf{P} = \frac{100 \times 60}{10 \times 3} = \frac{6000}{30} = \text{£}200.$$

E.g. 3 Find the rate % per annum simple interest at which £200 would have to be invested for 3 years to earn £60 interest.

$$\mathbf{R} = \frac{100 \times 60}{200 \times 3} = \frac{6000}{600} = 10\%.$$

E.g. 4 Find the time it would take £200 to earn £60 interest if it is invested at 10% per annum simple interest.

$$\mathbf{Y} = \frac{100 \times 60}{200 \times 10} = \frac{6000}{2000} = 3 \text{ years}.$$

(ii) Compound Interest

For **simple interest**, the **total** number of years is used at once. However, for **compound interest**, we must take **one year** at a time, remembering to **add** the **interest** accumulated in the year to the starting **principal**, thereby **increasing** the **principal** at the end of each year.You may use the Simple Interest Formula for compound interest, if you take **Y = 1** and work out the interest for each year, *remembering to add in the interest to the principal as you go along.*

We shall now take the example considered earlier under simple interest and see the difference when we use compound interest instead.

E.g. £200 is invested for 3 years at 10% per annum *compound* interest. Find the amount at the end of the 3 years.

<u>1st Year</u>:

Principal	£200
+ Interest = 10% of £200	£ 20
Principal + Interest	£220

2nd Year:

Principal	£220
+ Interest = 10% of £220	£ 22
Principal + Interest	£242

3rd Year:

Principal	£242
+ Interest = 10% of £242	£ 24.20
Principal + Interest	£266.20.

The amount at the end of 3 years is, therefore, £266.20, compared with £260 when simple interest was used.

Using the method above, the work on compound interest problems can be laborious and time-consuming, particularly when the period of time in the problem is lengthy.
Again, there is a formula which can be used very conveniently:

Compound Interest Formula

A = Amount of money after Y years
P = Principal
R = Rate % per annum
Y = No. of Years
I = Interest

$$\boxed{A = P\left(1 + \frac{R}{100}\right)^{Y}}$$

Next we shall demonstrate the use of the Compound Interest Formula in answering the question posed in the example above.

$$A = P\left(1 + \frac{R}{100}\right)^{Y}$$

A = Amount after **3** years
P = **£200**
R = **10 %** per annum
Y = **3**

$$A = 200\left(1 + \frac{10}{100}\right)^3$$
$$\Rightarrow A = 200 \times 1.1^3$$
$$\Rightarrow A = 200 \times 1.331$$
$$\therefore A = £266.20$$ (as before).

Appreciation/Depreciation

When an asset increases in value over a period it is said to *appreciate* and, if it decreases in value, it is said to *depreciate*.

Appreciation is dealt with in the same way as compound interest. If the *rates* of increase *vary* from one year to the next, the longer method of calculation *must* be used.
The Compound Interest Formula may be used if the *rate* of increase *stays the same* throughout.

E.g.(i) A property appreciates by 20% in the first year following purchase, 10% in the second year and 5% in the third year.
If the value of the property at the time of purchase is £10,000, find its value after 3 years.
(We must use the longer method.)

<u>1st Year</u>:

Value	£10000
+ Appreciation = 20% of £10000	£ 2000
Value + Appreciation	£12000

<u>2nd Year</u>:

Value	£12000
+ Appreciation = 10% of £12000	£ 1200
Value + Appreciation	£13200

<u>3rd Year</u>:

Value	£13200
+ Appreciation = 5% of £13200	£ 660
Value + Appreciation	£13860.

The value of the property after 3 years is, therefore:
£13860.

E.g.(ii) A property appreciates by 5% per year for the first three years following purchase.
If the value of the property at the time of purchase is £10,000, find its value after 3 years.
(The formula may be used here, since the rate does not change.)

$$\mathbf{A \quad = \quad P\left(1 + \frac{R}{100}\right)^{Y}}$$

A = Amount after **3** years
P = **£10000**
R = **5 %** per annum
Y = **3**

$$\mathbf{A \quad = \quad 10000\left(1 + \frac{5}{100}\right)^{3}}$$

Þ **A** = **10000** ´ **1.05**$^{\mathbf{3}}$
Þ **A** = **10000** ´ **1.157625**

\ **A** = **£11576.25**.

Depreciation is treated in a similar way to compound interest, except that the amount of depreciation in each year is **subtracted** from the value of the asset at the start of the year.
If the *rates* of decrease *vary* from one year to the next, the longer method of calculation *must* be used.
The Depreciation Formula may be used if the *rate* of decrease *stays the same* throughout.

Depreciation Formula

$$\boxed{\mathbf{A \quad = \quad P\left(1 - \frac{R}{100}\right)^{Y}}}$$

E.g.(iii) A machine depreciates by 20% in the first year following purchase, 10% in the second year and 5% in the third year.
If the value of the machine at the time of purchase is £10,000, find its value after 3 years.
(We must use the longer method.)

<u>1st Year</u>:

Value	£10000
- Depreciation = 20% of £10000	£ 2000
Value - Depreciation	£ 8000

<u>2nd Year</u>:

Value	£ 8000
- Depreciation = 10% of £8000	£ 800
Value - Depreciation	£ 7200

<u>3rd Year</u>:

Value	£ 7200
- Depreciation = 5% of £7200	£ 360
Value - Depreciation	£ 6840.

The value of the machine after 3 years is, therefore:

£ 6840.

E.g.(iv) A motor-cycle depreciates by 5% per year for the first three years following purchase.
If the value of the motor-cycle at the time of purchase is £10,000, find its value after 3 years.
(The formula may be used here, since the rate does not change.)

$$\mathbf{A} \quad = \quad \mathbf{P\left(1 - \frac{R}{100}\right)^{Y}}$$

A	=	Amount after **3** years	
P	=	**£10000**	
R	=	**5 %** per annum	
Y	=	**3**	

$$\mathbf{A} \quad = \quad \mathbf{10000\left(1 - \frac{5}{100}\right)^{3}}$$

Þ **A** = **10000** ´ **0.95^{3}**

Þ **A** = **10000** ´ **0.857375**

\ **A** = **£8573.75.**

EXERCISE 20

1. Copy the following table and fill in the gaps:

PRINCIPAL	RATE	TIME	SIMPLE INTEREST	
£ 200	20%	1.5 YRS	(_______________)	(a)
£ 140	25%	6 MTHS	(_______________)	(b)
£ 800	10%	(______)	£ 320	(c)
(____)	15%	1YR4MTHS	£ 40	(d)
£ 650	(____)	9 MTHS	£ 78	(e)

2. What is the simple interest if £2,000 is invested for three years at 4% interest per annum?

3. What sum of money must be invested at 4% simple interest to earn interest of £240 in three years?

4. What sum is accumulated if £5,000 is invested at 9% per annum compound interest for eight years?

5. What is the compound interest if £8,000 is invested for three years at 5% for the first year, 8% for the second year and 10% for the third year?

6. A property appreciates by 10% each year for the first three years after purchase. If the value at the time of purchase is £20,000, find the value at the end of the three years. (Answer to nearest £1000.)

7. A watch appreciates by 5% in the first year, 10% in the second year and 15% in the third year following purchase. If the watch costs £300, find its value after three years. (Answer to nearest £10.)

8. A car depreciates by 20% in its first year, 18% in its second year and 15% in its third year. Find its value after three years, if it costs £16,000 new. (Answer to nearest £100.)

9. A machine depreciates by 25% each year. If the value of the machine is £4,000 at present, what will be its expected value after five years? (Answer to nearest £10.)

10. The elephant population in a certain region is decreasing by approximately 5% each year. An estimate of the present number is 4,000. What would be the expected number in ten years' time? (Answer to nearest 100.)

EXERCISE 20 - ANSWERS

1. (a) £60 (b) £17.50 (c) 4yrs
   (d) £200 (e) 16%

2. £240

3. £2,000

4. £9,962.81

5. £1,979.20

6. £27,000

7. £400

8. £8,900

9. £950

10. 2,400